

International Commission on Mathematical Instruction

http://www.mathunion.org/ICMI

The Twentythird ICMI Study: Primary Mathematics Study on Whole Numbers

http://www.umac.mo/fed/ICMI23/

Discussion Document

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1. Introduction and Rationale for ICMI Study 23

This document announces a new Study to be conducted by the International Commission on Mathematical Instruction. This Study, the twentythird led by ICMI, addresses for the first time mathematics teaching and learning in the primary school (and pre-school), taking into account international perspectives, socio-cultural diversity and institutional constraints. One of the challenges of designing the first ICMI primary school Study of this kind is the complex nature of mathematics at the early level. For this reason a focus area has been chosen, as central to the discussion, with a number of questions connected to it. The broad area of **Whole Number Arithmetic (WNA)** including operations and relations and arithmetic word problems form the core content of all primary mathematics curricula. The Study of this core content area is often regarded as foundational for later mathematics learning. However, the principles and main goals of instruction in the foundational concepts and skills in WNA are far from universally agreed upon, and practice varies substantially from country to country. An ICMI Study that provides a meta-level analysis and synthesis of what is known about WNA would provide a useful base from which to gauge gaps and silences and an opportunity to learn from the practice of different countries and contexts.

Whole numbers are part of everyday language in most cultures, but there are different views on the most appropriate age at which to introduce whole numbers in the school context. Whole numbers, in some countries, are introduced in the pre-school, where the majority of children attend before the age of 6 years. In some countries, primary schooling includes Grades 1-6; in others it includes Grades 1-5. Thus the entrance age of students for primary school may vary

from country to country. For these reasons, this Study addresses teaching and learning WNA from the early grades, i.e., the periods in which WNA is systematically approached in the formal school, and in some contexts this includes the pre-school.

In January 2014, the International Programme Committee (IPC) for ICMI Study 23 met at the International Mathematical Union Secretariat, Berlin, and agreed upon four principles.

First it was decided that **cultural diversity** and how this diversity impinges on the early introduction of whole numbers would be one major focus. The IPC agreed that the Study will seek contributions from authors representative of as many countries as possible, especially those where cultural characteristics are less known but where these influence what is taught and learned. In order to foster an understanding of the different contexts where potential authors have developed their studies, each applicant for the Conference will be required to provide background information about this context.

Second, it was decided to find better ways to involve **policy makers** who have the responsibility to offer to every child the opportunity to go to school and to learn WNA. In connection with this aim, the IPC will also solicit contributions in the form of annotated videoclips about practical examples of WNA with potentially strong impact.

Third, it was decided to collect examples of experiences about **inclusive** teaching and learning, including students with special needs, considering that in some countries the education system provides special schools, classrooms and teachers whilst in others students are enrolled in mainstream classes.

Fourth, it was decided to focus on **teacher education and professional development**, considering that in order to teach elementary mathematics effectively there is a need for sound professional knowledge, both in mathematics and in pedagogy.

In order to meet this complex set of principles, the IPC delineated a set of **themes** to serve as the organizing framework for the Study Conference.

This discussion document presents the background of the Study, together with its challenges and aims and provides a description of the five organising themes. Because the Study Conference will be organized around discussion within each theme (with some overarching sessions) each proposed contribution to the Study should address the theme that it is most aligned with, and identify a second theme that it may also be related to. Finally the discussion document outlines the organisation, timing and location of the Study Conference and the timetable of the milestones leading up to the Study Conference and to ICMI publication.

1.1. Background of the Study

Countries differ enormously in providing pre-school programs (UNESCO, 2010) which are especially important for children from disadvantaged families: actually the OECD (2011) has reported that, in general, participation in pre-school produces better learning outcomes in later years such as for fifteen-year-old students. Primary schooling is compulsory in most countries (in all Western countries), although there is considerable variation in the facilities, resources and opportunities for students. This is the uneven context where mathematics teaching and learning takes place.

Mathematics is a central feature of early education and the content, quality and delivery of the curriculum is of critical importance in view of the kinds of citizens each country seeks to produce.

In the international literature there are many contributions about primary school mathematics. In many cases, especially in the West, early processes of mathematical thinking, often observed in early childhood (i.e., 3-8 year-old children), are also investigated by cognitive and developmental psychologists. They sometimes study the emergence of these processes in

clinical settings, where children are stimulated by suitable models so as to observe the emergence of aspects such as one-to-one correspondences, counting, measuring and other processes). In several countries, Piaget's theory has been very influential despite criticism. Neuroscientists have also been studying for some years the emergence of "number sense". However, recent perspectives highlight that what is still needed is serious and deep interdisciplinary work with experts in mathematics education (UNESCO, 2013).

1.2. Key Challenges for ICMI Study 23

A recent document prepared by ICMI's Past President Michéle Artigue and commissioned by UNESCO (2012) discusses, from a political perspective, the main challenges in fundamental mathematics education. It reads:

We live in a world profoundly shaped by science and technology. Scientific and technological development has never been faster, has never had an impact as important and as immediate on our societies, whatever their level of development. The major challenges that the world has to face today, health, environment, energy, development, are both scientific and human challenges. In order to take up these challenges, the world needs scientists able to imagine futures that we barely see and able to make these possible, but it also needs that the understanding of these challenges, the debate on the proposed changes, are not reserved for a necessarily limited scientific elite, but are very widely shared. Nobody can now doubt that positive, sustainable and equitable evolutions cannot be achieved without the support and contribution of the great majority of the population. Nobody should thus doubt that the gamble of shared intelligence, that of quality education for all, and especially science education for all, including mathematics and technology education, are the only gambles we can take. This is even more the case in the current context of crisis. Without such an education, it is futile to speak of debate and citizens' participation.

Drawing on this ideas, ICMI has acknowledged that it is timely to launch, for the first time in its history, an international Study that especially focuses on early mathematics education, that is both basic and fundamental mathematically. Primary school mathematics education has been present in other ICMI studies, but, in most cases, secondary school mathematics education was predominant. When foundational processes are concerned, a strong epistemological basis is needed. This is where the involvement of ICMI adds value with respect to analyses carried out in other fields. Such epistemological analysis was part of classical works of professional mathematicians (e.g., Klein, Smith, and Freudenthal) who played a major role in the history of ICMI and considered mathematics teaching as a whole (ICMI, 2008). It is pertinent to mention here a short quote by Felix Klein, the first President of ICMI, used as an epigraph in the website on the history of ICMI (ICMI, 2008).

I believe that the whole sector of Mathematics teaching, from its very beginnings at elementary school right through to the most advanced level research, should be organised as an organic whole. It grew ever clearer to me that, without this general perspective, even the purest scientific research would suffer, inasmuch as, by alienating itself from the various and lively cultural developments going on, it would be condemned to the dryness which afflicts a plant shut up in a cellar without sunlight (Felix Klein, 1923).

One cannot study school mathematics teaching without focusing also on the teacher's role and responsibility. The attention on mathematics teacher education and professional

development has been a constant preoccupation of ICMI. The primary school and (more generally) early education deserves a special attention. The complex nature of arithmetic and its foundational importance for mathematics are well known by mathematicians and mathematics educators. However, primary school teachers work within systems which may or may not support a rigorous professional environment in which they are knowledgable and respected experts on both the mathematics and the pedagogy of what they teach. In some systems, teaching WNA may be treated as something that virtually any educated adult can do with little specific training; WNA may be viewed by some as straightforward and intuitive, and involving no more than showing children how to cope with everyday life and to carry out algorithms.

There are systems where primary mathematics teachers are specialists and other where they are generalists. It is not within the aims of this Study to enter deeply into the pedagogical debate about specialist vs generalist teachers in early education, as both models show advantages and disadvantages. What is important to highlight is that much is already known from research about productive ways to teach WNA, yet this knowledge cannot be enacted in systems in which teachers are not proficient in elementary mathematics and particular pedagogical approaches. Effective teacher education may require the development of a culture in which teachers are expected to be highly educated professionals.

2. Aims of the ICMI Study 23

This Study aims to produce and share knowledge about sustainable ways of promoting effective pedagogy in teaching and learning WNA for all, taking account of the large body of theory and research already existent, socio-cultural diversity and institutional constraints. In particular the following specific aims were acknowledged by the IPC, for the early teaching and learning of WNA:

- bring together communities of international scholars representative of ICMI's diverse membership across regions and nationalities in addressing the theme of WNA for the production of a Study volume;
- provide a state-of-the-art expert reference group on the theme of WNA;
- > contribute to knowledge, better understanding and resolution of the challenges that teaching and learning WNA faces in diverse contexts;
- > collectively represent the wide variety of concerns in the field of WNA and reflect upon it;
- facilitate multi- and interdisciplinary approaches (including cooperation with other bodies and scientific communities) to advance research and development in WNA;
- disseminate scholarship in mathematics education research; methodologies, theories, findings and results, practices, and curricula in the theme of WNA;
- pave the way to the future by identifying and anticipating new research and development of WNA;
- > act as a resource for researchers, teacher educators, policy and curriculum developers and analysts and the broad range of practitioners in mathematics and education;
- promote and assist discussion and action at the international, regional or institutional level.

3. The Themes of the ICMI Study 23

The ICMI Study will be organised around five themes that provide complementary perspectives on approaches to early WNA in mathematics teaching and learning. Contributions to

the separate themes will be distinguished by the theme's specific foci and questions, although it is expected that interconnections between themes will emerge and warrant attention.

The five themes are:

- 1. The why and what of WNA
- 2. Whole number thinking, learning, and development
- 3. Aspects that affect whole number learning
- 4. How to teach and assess WNA
- 5. Whole numbers and connections with other parts of mathematics.

Themes 1 and 2 address foundational aspects from the cultural-historic-epistemological perspective and from the (neuro)cognitive perspective. What is especially needed are reports about the impact of foundational aspects on practices (both at the micro-level of students and classrooms and at the macro-level of curricular choices).

Themes 3 and 4 address learning and teaching respectively, although it is quite difficult, sometimes, to separate the two aspects, because for example in some languages and cultures (e. g., Chinese, Japanese, Russian) the two words collapse into only one.

Theme 5 addresses the usefulness (or the need) to consider WNA in connection with (or as the basis for) the transition to other kinds of numbers (e.g., rational numbers) or with other areas of mathematics, traditionally separated from arithmetic (e.g., algebra, geometry, modelling).

Each theme is outlined briefly and followed by exemplary questions that could be addressed in the submitted contributions. An overarching question which cuts across all the themes concerns teacher education and development:

• How can each of the themes be effectively addressed in teacher education and professional development?

3.1. The why and what of WNA

This theme will address cultural-historic-epistemological issues in WNA and their relation to traditional, present and possible future practices.

A sense of number is constructed through everyday experience, where culture and language play a major role, hence ethnomathematics has paid attention to the different grammatical constructions used in everyday talk (e.g., Maori numbers as actions; Aboriginal Australians' spatial approach to numbers). Ways of representing whole numbers and making simple calculations (e.g., with fingers or other body parts; with words; with tools, including mechanical and electronic calculators; with written algorithms) have enriched the meaning of whole numbers through the ages.

The base ten system is critical for our current sophisticated understanding of WNA. The long and difficult development of place value systems is well documented in the history of mathematics (the introduction of place value in China and India; the migration to Europe through the Arabic culture; the invention of zero; the strategies for mental calculation) and indicates the need to study place value and the base ten system deeply for understanding.

The above issues (and others) have been considered in different ways by different cultures throughout history. Besides the use of numbers in practical activities, there is evidence (in the history and in educational research) that the exploration of the properties of whole numbers, relations and operations paves the way towards the introduction, with young students too, of typical mathematical processes, such as generalizing, defining, arguing and proving.

Some references may be found in the ICMI Studies 10, 13, 16, 19.

- What goals underlie the teaching and learning of WNA?
- Taking a mathematical perspective (as practised by the current community of mathematicians) combined with an educational perspective, what are core mathematical ideas in developing pathways to WNA?
- What are distinctive features concerning whole number representation and arithmetic in your culture? What is the grammar of number? In what ways does language or ways of representing and using numbers influence approaches to calculation or problem solving? How do these features interact with the decimal place value system?
- What is the role of mathematical practices and habits of mind in teaching and learning WNA? How can teaching and learning WNA support the development of mathematical practices and habits of mind?
 - How much is the base ten place value emphasized in your curriculum?
- How much computational facility is important for later mathematics learning, and learning in other areas? What about mental calculation? What about speed of calculation?
- How do policies and the educational environment and system support or not support a culture in which teaching WNA is seen as requiring detailed, specific professional knowledge?
- What were the main historic features and their origins of WNA in (ancient) west / east? What were some factors that led to such historic features? What were the effects on the development of mathematics curriculum?
- How does your curriculum develop understanding of the structural features of WNA and its extensions?

3.2. Whole number thinking, learning, and development

This theme will address the relationships between cognitive and neurocognitive issues and traditional, present and possible future practices in the early teaching and learning of WNA.

The idea of number sense was in use for decades in the mathematics education literature before entering into the cognitive and neurocognitive literature, with some similarities and differences. (Neuro)cognitive scientists have investigated children's spontaneous tendency to focus on numerosity in their environment; the development of rapid and accurate perception of small numerosities (subitizing) in connection with visualization and structuring processes; the ability to compare numerical magnitudes; and the ability to locate numbers on a (mental) number line. There are models for children's informal knowledge of counting principles and informal counting strategies and their development into more formal and abstract arithmetic notions and procedures.

A recent focus concerns developmental dyscalculia, as a difficulty in mathematical performance resulting from impairment to those parts of the brain that are involved in arithmetical processing, without a concurrent impairment in general mental function.

Recent debates concern the embodied cognition thesis resulting in the evidence, shared by many researchers, that, although mathematics may be socially constructed, this construction is rooted in, and shaped by, the body and bodily experiences.

Some references may be found in OECD (2010), UNESCO (2013).

- To what extent is basic number sense inborn and to what extent is it affected by socio-cultural and educational influences? How is the relationship between these precursors/foundations of WNA, on the one hand, and children's WNA development?
- What can we learn from the (neuro-)cognitive studies in WNA? Do their findings essentially confirm insights that are present (and were already present for a long time) in the

mathematics education community or do they point to truly new insights and recommendations about the kind of tasks and instructional approaches children need? How can we integrate different perspectives about the foundations and development of WNA concepts and skills?

- What are specific effects of the structure of the individual finger counting system on mental and linguistic quantity representation and arithmetic abilities in children, and even in older learners and adults?
- How can an embodiment framework be used to analyse and/or design educational approaches based on suitable representations, (e.g., through the number line) or on-line manipulatives and modern technological devices (touchscreens)?
- What are appropriate ways of analyzing the multimodal nature of mathematical thinking (e.g., the role of bodily motion and gesture)?
- What is the relationship between the embodied cognitive approach and traditional approaches, for example Montessori, Piaget, which had a strong influence of elementary school mathematics worldwide?
- How can the tools of the embodiment framework/analysis be integrated/combined with socio-cultural perspectives to compare/contrast approaches where embodiment is exploited or hindered?
- How can teachers be educated in order to exploit the (neuro)cognitive foundations for WNA?

3.3. Aspects that affect whole number learning

This theme will address some aspects affecting learning of WNA in both positive and negative ways.

Socio-cultural aspects influence enumeration practices, algorithms and representations as well as metaphors or models (e.g., the number line). Hence students' language and culture may help or hinder the construction of WNA not only in schools but also in informal settings. On the one hand, the recourse to tools from the history of mathematics (e.g. counting sticks; different kind of abaci; reproduction of ancient mechanical calculators) may be effective to foster learning of WNA with explicit reference to the local culture. On the other hand, intentionally designed tools may address the effective learning processes evidenced in the literature (e.g., technological tools including the multitouch ones).

Low achievement in WNA is a major focus in debates at all levels, from school practice to international studies. Literature shows that it may depend on very different aspects: context variables (e.g., marginalized students; migrant and refugee students; education in fragile democracies), institutional variables (e.g. different languages in school and out of school context), learning disabilities (dyscalculia; sensual impairment for deaf and blind students); affect factors (e.g., self-beliefs, anxiety, motivation, gender issues); didactical obstacles (e.g., a too limited approach as in the case of teaching addition separate from subtraction or multiplication as a repeated addition only); or epistemological obstacles (related to the historical process of constructing WNA by mankind).

Some references may be found in the ICMI Studies 17, 22 and, for general issues concerning the contexts, UNESCO (2010).

The following possible questions will help to illuminate this theme further:

• What are the features of your language related to whole numbers, operations and word problems that could affect learning in a positive or negative way? How these features are mirrored in formal or informal settings?

- What main challenges for learning WNA are faced by marginalized students or, in general, in difficult contexts?
- What main challenges are faced for learning WNA by students with sensual impairments (blind and deaf)?
 - What main challenges are faced for learning WNA by dyscalculic students?
- In your country are students with special needs enrolled in mainstream classes (inclusive systems) or in special education classes? To what extent may the strategies for learning WNA especially developed for students with special needs be useful for all students?
- In your country is there evidence that the literature on either didactical or epistemological obstacles has impact on classroom practice?
- Which tools (from the ancient or new technologies) are useful to enrich classroom activity for all or to help low achievers in WNA? Is there evidence on effective use of traditional manipulatives (including the ones rooted in local cultures), virtual manipulatives, technologies (including the recently developed multi touch technologies)? Are there classroom studies on the comparison of different kinds of tools?
 - What strategies may be implemented by teachers in relation to the above issues?

3.4. How to teach and assess WNA

This theme will address general and specific approaches to the teaching, assessing and learning of WNA. WNA appears in standards documents for mathematics of every country (see http://www.mathunion.org/icmi/other-activities/database-project/introduction/), and in specific international studies (e.g. the Learner's perspective Study, with sixteen country teams). In some countries independent research communities have also developed projects on teaching and assessing WNA, which, in some cases, are internationally acknowledged (e.g. Realistic Mathematics Education in the Netherlands; NCTM Curriculum and Evaluation Standards in US; Davydov's math curriculum in Russia; the Theory of Didactical Situations in France). In the ethnomathematics trend, projects sensitive to the local cultures and traditions have been developed (e.g. in Australia, Latin America, USA and Canada). A specialized Symposium on Elementary Mathematics Teaching (SEMT) has been held every second year in Prague since 1991.

Some other focus issues may be the following: the role of textbooks and future teaching aids (e.g., multimedia; e-books) for WNA; tools to approach specific elements of WNA (e.g., manipulatives, technologies); specific strategies for some fields (e.g., for word problems, the Chinese tradition of problems with variation; Singapore's model method; the extended literature on word problems and relations with real life situations); examples of practices rooted in local culture; metacognitive aspects in national curricula (e.g. early approaches to mathematical thinking processes).

In recent years the assessment debate at the local and school level has been very much biased by the results of international studies (e.g. OECD PISA, TIMSS), which are likely to produce assessment driven curricula. An ICMI Study on assessment was produced in the early 90s (ICMI Study 6), but updating might be necessary to establish current relevance and the impact of the international studies.

Some references for this theme may be found in the proceedings of ICMI Congress and Regional Conferences http://www.mathunion.org/icmi/Conferences/introduction/.

- What are the consequences of policy decision making related to evidence-based WNA teaching in comparison with policy decision making based on opinion?
 - How is the intended curriculum reflected in textbooks and other teaching aids?

- What are the changes (if any) that have resulted from the use of technology in the teaching of WNA?
- How complety is our understanding of the development of the place value system, and at what points in the/your curriculum are key features of place value explored in greater depth?
- How does the/your curriculum foster the transition from a counting or additive view of number to a ratio/multiplicative/measurement view of number?
- How do children acquire WNA concepts and procedures outside of school? How can teachers build upon the knowledge children acquire outside school?
- What are the approaches that have proven to be effective in your school setting to teach elements of WNA, for example number sense, cardinality, ordering, operations (subtraction with re-grouping, etc.), problem solving, estimation, representing, mental computation...?
- Problem-solving contexst: should they be realistic? Should they be authentic? Always? What is the place (if any) of traditional word problems? What is the role of (real world) context in WNA? Are they always necessary?
 - How can we develop positive attitudes toward mathematics while teaching WNA?
- How can teachers promote the development of student's metacognitive strategies during the learning of WNA?
 - What main challenges are faced by teachers when teaching and assessing WNA?
- What innovative assessment approaches are used to evaluate the learning outcomes of WNA? What are the changes (if any) in assessment of WNA that have resulted from the media appeal of international studies like PISA or TIMSS?

3. 5. Whole numbers and connections with other parts of mathematics

This theme will address WNA in terms of its interrelationships with the broader field of mathematics.

Some connections are of central concern: pre-algebra and algebraic thinking (e.g. looking for patterns; schemes for the solution of world problems); geometry or spatial thinking (e.g., triangular or square numbers and similar; number lines); rational numbers and measurement (e.g. Davydov's curriculum for arithmetic); statistical literacy (e.g. mean, median and mode, interval, scale, and graphical representation).

Evidence suggests that the earliest formation of WNA can support the learning of mathematics as a connected network of concepts and, viceversa, embedding WNA in the broad field of mathematics can foster a better understanding.

Some references for this theme may be found in the ICMI Studies 9,12,14,18.

- How can WNA teaching and learning contribute to understanding other interconnected mathematical ideas and build on one another to make students view mathematics as a coherent body of knowledge?
- In your country, to what extent are connections between WNA and other Mathematics topics pointed out in the curriculum syllabus and textbooks, and how are they approached? i.e WNA and measurement, WNA and elementary statistics? Pre-algebra patterns, WNA and algebra?
- In your system/country are symbolic and non-symbolic approaches to word problems compared? To what extent are connections made between base ten arithmetic and polynomial arithmetic? To what extent are the rules of arithmetic/properties of operations used as a guide in learning manipulation of algebraic expressions?

- In your country/system to what extent are connections between WNA and other Mathematics topics stressed in the teachers' education programs?
- In what ways does the connection between WNA and specific themes in other areas of Mathematics contribute to students' understanding of these themes?
- What learning conditions enable students to make connections between WNA and other mathematics topics?
- In which ways do the practice of connecting WNA to other areas of mathematics contribute to the development of mathematical thinking?
- How does the connection of WNA with other areas of mathematics improve communication of mathematical ideas?
- How can technology be used to make connections between WNA and other mathematics topics?
- How does the use of representations in WNA teaching and learning contribute to building connections with other mathematical areas? For example, to what extent is the number line used to exhibit the connections between WNA and arithmetic of fractions?

4. The Study Conference

ICMI Study 23 is designed to enable teachers, teacher educators, researchers and policy makers around the world to share research, practices, projects and analyses. Although reports will form part of the program, substantial time will also be allocated for collective work on significant problems in the field, that will eventually form part of the Study volume. As in every ICMI Study, the ICMI Study 23 is built around an International Conference and directed towards the preparation of a published volume.

The Conference will be organized around working groups on the themes: these groups will meet in parallel during the time of the Conference. In each working group, the IPC will organise the discussion starting from the contributions, assuming that each participant has carefully reviewed the contributions of their working group. Some special sessions presenting video-clips of practice will be organized, to share meaningful examples of WNA. Thus, there will be plenty of time for discussion of submitted papers, as well as possible plans for future collaborative activity.

The Conference language is English. However, native speakers and more expert participants will do every effort to ensure that every participant may take active part in the discussion.

4.1. Location and dates.

The Study Conference will take place in Macau, China and will be hosted by the University of Macau (June 3-7, 2015), with an opening on June 3 at 9AM and closing on June 7 at 2PM. Arrival day is June 2; departure may be scheduled as from the night of June 7.

Every effort will be made to assist participants with visa applications, if needed.

4.2. Participation

As is the usual practice for ICMI studies, participation in the Study Conference will be by invitation only for the authors of submitted contributions which are accepted. Proposed contributions will be reviewed and a selection will be made according to the quality of the work, the potential to contribute to the advancement of the Study, with explicit links to the themes

contained in the Discussion Document and the need to ensure diversity among the perspectives. The number of invited participants will be limited to approximately 100 people.

Unfortunately, an invitation to participate in the Conference does not imply financial support from the organizers, and participants should finance their own attendance at the Conference. Funds are being sought to provide partial support to enable participants from non-affluent countries to attend the Conference, but it is unlikely that more than a few such grants will be available. Further information about the access to such grants will be available in the ICMI Study 23 website

http://www.umac.mo/fed/ICMI23/

4.3. ICMI Study 23 Products

The **first product** of the ICMI Study 23 is an electronic volume of Proceedings, to be made available first on the Conference website and later in the ICMI website: it will contain all the accepted papers as reviewed papers in a Conference Proceedings (with ISBN number).

The **second product** is a gallery of commented videoclips about practices in WNA, to be hosted in the Conference website and, possibly, later, in the ICMI website.

The **third product** is the ICMI Study volume. The volume will be informed by the papers, the videoclips and the discussions at the Study Conference as well as its outcomes, but it must be appreciated that there will be no guarantee that any of the papers accepted for the Study Conference will appear in the book. The Study book will be an edited volume published by Springer as part of the New ICMI Study Series. The editors and the editing process and content will be the subject of discussion among the IPC considering also the framework prepared for the Study Conference. It is expected that the organization of the volume will follow the organization and themes of this Discussion Document, although some changes might be introduced to exploit the impact of the discussion raised during the Conference. A report on the Study and its outcomes will be presented at the 13th International Congress on Mathematical Education, to be held in Hamburg, Germany (24-31 July 2016). It is hoped that the Study volume will also be published in 2016.

5. Call for Contribution to ICMI Study 23

The IPC for ICMI Study 23 invites submissions of contributions of several kinds: theoretical or cultural-historic-epistemological-essays (with deep connection with classroom practice, curricula or teacher education programs); position papers discussing policy and practice issues; discussion papers related to curriculum issues; reports on empirical studies; video-clips on explicit classroom or teacher professional education practice. The possibility of submitting short video-clips is a novelty of the ICMI Study 23. Video-clips show in a visual way examples of non-verbal communication, dynamic moments of significance or oddity, impressive performances or crucial incidents about teaching and learning WNA (including teacher professional education and development). Hence, in addition to usual reports, video-clips with accompanying short paper (see below) are welcome.

To ensure a rich and varied discussion, participation from countries with different economic capacity or with different cultural heritage and practices is encouraged.

The IPC encourages people who are not familiar with such Conferences to submit early (see the deadlines below) in order to receive assistance for finalizing their contribution (this assistance concerns the choice of the topic of the contribution and the structure of the paper, not the editing of English language). In this way the IPC inaugurates a new tradition of helping newcomers (including practitioners) to the international mathematics education community. This implies a

process of supporting the writing of a contribution which the IPC judges as having the potential to contribute to the Study (see below).

An invitation to the Conference does not imply that a formal presentation of the submitted contribution will be made during the Conference or that the paper will appear in the Study volume published after the Conference.

5.1. Submissions

The ICMI Study 23 website is opened at the address

http://www.umac.mo/fed/ICMI23/

The website will be regularly updated with information about the Study and the Study Conference and will be used for sharing the contributions of those invited to the Conference in the form of Conference pre-proceedings.

Two kinds of submissions are welcome:

papers prepared in English (the language of the Conference) according to a **template** (max 8 pages).

video-clips (5-8 minutes) with English subtitles together with an accompanying paper prepared according to a template (max 6 pages) together with the author's declaration of having collected informed consent forms signed by the participants. The English subtitles are required also in the videos with English speakers, in order to help the understanding of the interaction for non native speakers. Blurring faces of participants for privacy reasons, when needed, must be ensured by the applicants before sending the videos.

The files are to be saved with the name

Familyname_name

Accepted file extensions are the following:

papers: .doc; .docx; .odt together with a .pdf copy.

videos: .mp4; 3gp.

In both cases, the indication of the working group - theme (1st and 2nd choice) where the paper or the video-clip is expected to be discussed must be included.

In both cases, also the **context form** has to be filled out by all the author(s) as completely as possible to help readers to understand the context of the contribution and interpret the contribution accordingly.

The template, the context form and the informed consent form will be available in the ICMI Study 23 website.

It is not allowed to submit two papers with the same first author.

Information about the technical way of submitting a paper or a video+paper will be available soon in the ICMI Study 23 website.

http://www.umac.mo/fed/ICMI23/

5.2. Deadlines

August 31, 2014: People who believe they need assistance for finalizing their contribution must submit a tentative copy with an appropriate form (**assistance form**) for requiring assistance no later than August 31, 2014. Their submissions will be examined immediately. The author will receive by September 30 the information of the decision (rejected, accepted pending revision, accepted in the present form). In the second case an IPC member will act as "tutor" to help the final preparation of the paper. Then the final paper will undergo the standard review process. The assistance form will be available in the ICMI Study 23 website.

September 15, 2014: Submissions by people who do not require assistance must be sent no later than September 15, 2014, but earlier if possible.

February 2015: Proposals will be reviewed, decisions made about invitations for the Conference (to be held in June 2015) and notification of these decisions sent by the end of February.

Information about visa, costs and details of accomodation will be available on the ICMI Study 23 website

http://www.umac.mo/fed/ICMI23/

Further information may be asked at the following address:

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7. References

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