

# CULTURAL ISSUES IN THE COMMUNICATION OF RESEARCH ON MATHEMATICS EDUCATION

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Although culture “permeates all aspects of educational endeavor and should be acknowledged more explicitly than it is” (Andrews, 2010, p. 3), awareness of the effects of cultural diversity and of ways to benefit from it in research on mathematics education is rather new (Artigue, 2008). Internationally, this awareness may be traced back to research comparing Western and Eastern traditions in mathematics education. International attention was prompted by the astonishing performance of Eastern students in international mathematics assessments. Up until the 1970s, little was known in the West about China’s education system, for example, but in recent decades, more and more information has been disseminated about mathematics teaching and learning in China and about the background culture (see, for example, Leung *et al.*, 2006). Research publications on this topic usually include a large section on the cultural background, because of the need to reconstruct for outsiders (often, reluctant outsiders, accustomed to consider their system of values and beliefs as the only possible one) the conditions and constraints given by the context (for example, Sun, 2011, where the quantity of “cultural” footnotes is in stark contrast with the standard format of the journal). In contrast, because of a naively “taken-as-shared” system of values, a researcher from the West, *centered* in his/her own world, does not feel obliged to reconstruct the cultural background for where the study has been conducted. So how is cultural background related to mathematics teaching and learning?

A useful schema is presented by Xie & Carspecken (2008): the intended, implemented and attained mathematics curriculum implicate philosophical assumptions coming from the cultural background that varies between nations (and perhaps regions) and that depend on historical traditions (see Figure 1). Underlying the processes which can be observed and investigated in schools (in the top part of the schema), there are many implicit or explicit assumptions, some related to the image of mathematics and some related to more general worldviews, depending on the cultural background. Xie & Carspecken’s (2008) aim is to compare dialectical materialism and pragmatism in relation to Chinese and US mathematics curricula. Hence they focus on the markedly different philosophies in these two cultural traditions. Andrews (2010) considers a much finer grain, focusing on countries or regions that are not so far apart, like Flanders, England, Hungary, Spain and Finland. Andrews reviews several studies of the links between culture and mathematics curricula, and the

routines of mathematics lessons, and concludes with an appeal to researchers of mathematics teaching and learning “to make explicit in the reporting of their work the cultural context in which it was undertaken” (Andrews, 2010, p. 12).

An additional problem is raised by Adler *et al.* (2005): “research in countries where English is the national language dominates the literature” (p. 372). Researchers from non-Anglophone countries, like the two authors of this article, have to struggle against the constraints of language. Language can create differences in academic discourse and barriers in academic communication (for a discussion, see Duszak, 1997; Fløttum, 2007; Siepmann, 2006). The effect of this communication problem (and language discrimination) is the impoverishment of the whole community of researchers in mathematics education (Bishop, 2008).

The question of cultural background applies to every study in mathematics education and, in particular, to studies about mathematics teacher education. There are more and more projects researching mathematics teacher education, but the cultural background of such projects is not often reported. In order to compare large-scale projects involving national reform initiatives in Austria, Ohio, South Korea and Australia, Pegg and Krainer (2008) use the following framework: they compared the initial impetus for the initiative, goals and intervention strategies, implementation and communication, evaluation and impact, challenges and future steps. We believe that it is also necessary to explain in more depth how the research design and implementation is related to the cultural background: the results and success (if any) of the project may depend on implicit values which are not likely to be found in other contexts.

In this article, we present an example of a mathematics teacher education project developed by our research team at the Mathematical Machines Laboratory. The MMLab-ER project draws on research developed since the early 1990s. Our presentation of the project illustrates how the cultural background has influenced both the design of activities and the choice of topics used in the study. Following a summary of the cultural background, we give short outline of the project itself, using the framework adopted by Pegg and Krainer (2008).

**An example of a local project on mathematics teacher education: the MMLab-ER project [1]**  
The MMLab-ER project is being conducted by the Mathematical Machines Laboratory [2], of the University of

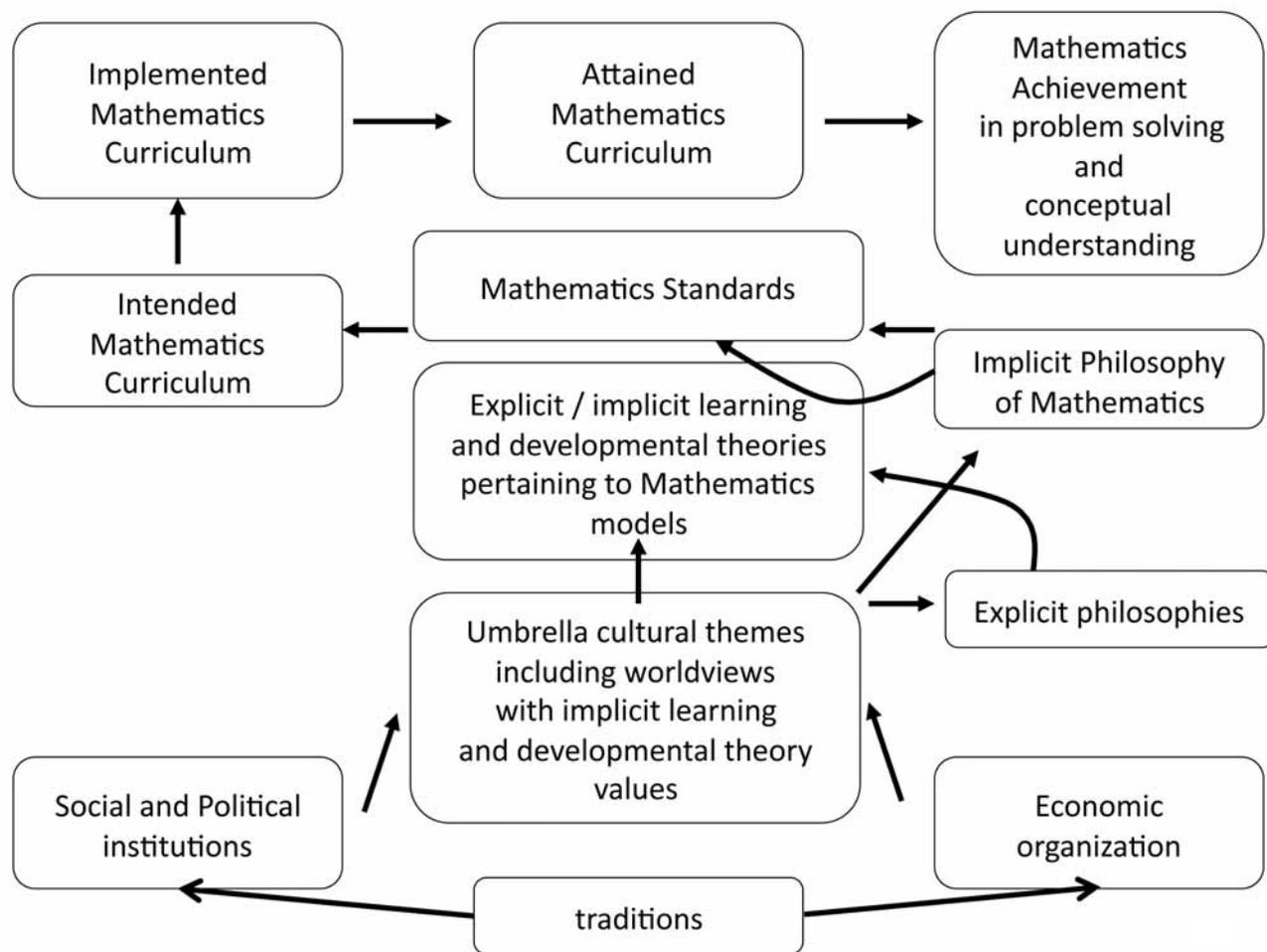


Figure 1. The complexity of relationships in mathematics education (adapted from Xie & Carspecken, 2008, p. 17).

Modena and Reggio Emilia, Italy. The Laboratory is a research centre for the teaching and learning of mathematics by means of instruments (Ayres, 2005). The name comes from the most important collection of the Laboratory, the *Mathematical Machines*, which are working reconstructions of mathematical instruments taken from the history of mathematics. Most of the machines concern geometry: “a mathematical (geometrical) machine is a tool that forces a point to follow a trajectory or to be transformed according to a given law” (Bartolini Bussi & Maschietto, 2006). The simplest mathematical machine is the pair of compasses; there are also more complex curve-drawing devices and pantographs to represent geometric transformations [3]. The major focus on geometry is consistent with Italian curricula, in which geometry is a significant presence from 1st grade to 13th grade (students aged 6-18 years).

A typical teaching experiment involving mathematical machines lasts several sessions, from several weeks to a few months (for a review of the published studies, see Bartolini Bussi, 2009). These experiments, which rely on cooperation with teachers, are made possible by the ongoing involvement of the same mathematics teacher with the same

group of students for many years; in Italy, classes are usually taught by the same teachers for 5 years in primary school, for 3 years in junior secondary school and for at least 2 years (sometimes 3 or 5 years) in secondary school. These extended periods with the same group of students make teachers less anxious about the short term effects of their teaching and encourages them to take care of and to observe long term processes.

The *mathematical laboratory* is an important construct that characterizes the activities carried out in these teaching experiments. The institutional history of this construct may be traced as follows. In 2001-2004 the Italian Mathematical Union prepared three volumes (*Matematica 2001*, *Matematica 2003* and *Matematica 2004* [4]) on behalf of the Ministry of Education, proposing a mathematics curriculum for grades 1-13. *Matematica 2003* contains a special section on the idea of *mathematical laboratory* [5]:

The mathematical laboratory shows similarities with the concept of Renaissance workshops in which apprentices learned by doing and watching what was done, communicating with one another and with experts. The

construction of meanings within the mathematical laboratory is strictly linked to the instruments used when carrying out the given activities and to the interaction among the participants in the activities. It is important to remember that tools are the result of a cultural evolution; they have been produced with specific aims and, as such, embody ideas. This has important didactical implications: first of all, meanings do not live only in the tools and cannot emerge purely from the interaction of the pupils with the tools. Meanings are rooted in the aims for which the tools are used, and in the strategies related to the use of those tools that are elaborated in the course of the activities; moreover, the appropriation of the meanings requires individual reflection on the objects of study and the proposed activities. (pp. 60-61)

The following tools are listed as examples: “poor” materials (that is “everyday” traditional materials, such as squared paper, transparencies, folding paper), software and applications from information and communication technologies and mathematical machines (called “mechanisms” in the quoted document), the latter being considered as making possible experiences even “sometimes richer than the exploration carried out with dynamic geometry software” (p. 61). Historical sources, too, are tools in the mathematical laboratory, according to the Italian tradition of focusing on the history of ideas (in the Italian secondary curriculum, the history of subjects, such as the history of literature, the history of philosophy and so on, are in the foreground). Laboratory activities require communication that goes far beyond lecturing:

The construction of meaning is strictly linked to the communication and sharing of knowledge in the classroom, through collaborative or cooperative group work and through the mathematical discussion orchestrated by the teacher (p. 62).

This last theoretical construct (mathematical discussion) draws on the Vygotskian perspective on mathematics teaching and learning (Arzarello & Bartolini Bussi, 1998; Bartolini Bussi & Mariotti, 2008), supported by several translations into Italian of Vygotsky’s work and that of his students. These translations were produced earlier than in other Western countries and were not biased by ideological prejudices against marxism (Bartolini Bussi, 1994).

The idea of a mathematical laboratory is neither new nor original. Mathematicians have always used tools for theoretical purposes (*e.g.*, straightedge and compass; abaci and mechanical calculators). Mathematics teaching activities involving historical mathematical instruments appeared in the last century in many places, including Europe, the United States and Japan (Bartolini Bussi, Taimina & Isoda, 2010). In recent decades laboratory activities have been more focused on computer tools, but, as mentioned above, the Italian standards include also concrete tools and sources from the history of mathematics. In France (one of Italy’s neighbours), there is no reference to these kinds of laboratory activities in the 2008 standards for mathematics [6], and what is called in French the “*démarche d’investigation en mathématiques*” is still the subject of heated discussion [7].

In Italy, the active involvement of all students in mathematics laboratories also meets the needs of a totally inclusive system: uniquely in Europe, Italy’s National Law 118 (1971) and National Law 517 (1977) established inclusive education as national policy. All students are educated in ordinary classrooms, with specialized support as needed based on a student’s individualized education plan. The MMLab activities have proved to be useful and successful for low achievers in vocational schools and for visually impaired students [8].

Since the early 1990s, studies at MMLab have interlaced basic research and classroom research with research on pre-service and in-service teacher education: the latter became more and more relevant with the passing of time [9]. An important reference for us, in the context of mathematics teacher education, is the *Cultural Analysis of Content (CAC)*:

CAC adds to professional knowledge usually considered in the literature (“subject matter knowledge”, “pedagogical content knowledge”, and “general pedagogical knowledge”—see Shulman, 1986) the understanding of how mathematics can be arranged in different ways according to different needs and historical or social circumstances, and how it enters human culture in interaction with other cultural domains (economics, physical sciences, philosophy, *etc.*) (Boero & Guala, 2008, p. 223).

Boero and Guala claim that it is important to develop teachers’ mathematical knowledge, while calling into question teachers’ beliefs and ways of thinking about mathematics, drawing on historical, epistemological, cognitive and didactical issues. Accordingly, in the MMLab, prospective and practising teachers are encouraged to analyse mathematical machines as special tools used by mathematicians in the past to develop the defining and proving processes that characterize Western mathematical culture.

After this account of the cultural background to MMLab-ER, we provide an overview of the project using the framework proposed by Pegg & Krainer (2008).

### **Impulse for the initiative and challenge**

In the Italian curriculum standards, the idea of a mathematical laboratory is often mentioned [10]: it is, however, acknowledged [11] that educational design in a mathematical laboratory is not easy. Specific pre-service and in-service teacher education projects are therefore needed, a challenge that is far from being completely realized. The MMLab-ER project was founded in 2008 by the Emilia Romagna Region, in agreement with the Regional School Office. The project was designed to facilitate the implementation of a laboratory approach in the teaching and learning of mathematics, focusing mainly, but not exclusively, on geometry. The project was seen as a regional answer to the national needs discussed above. The challenge was to share the findings of MMLab research studies with teachers and to develop innovative activities in schools.

### **Goals and intervention strategies**

The goal of the project was agreed by the steering commit-

tee, in which researchers, teachers, policy makers and school administrators were represented. This goal was twofold:

- to create a network of mathematical laboratories in different provinces of the Emilia-Romagna region with a selected collection of mathematical machines, in order to reproduce the environment and activities developed in the Modena laboratory;
- to prepare a network of local groups of teachers to be able to implement a laboratory approach in their classrooms and to take on the organization and maintenance of local laboratories once the project had ended.

The participating teachers, who were selected by school principals, were mainly from secondary schools (from grade 6 to grade 13, that is, students aged 11-18 years). The pedagogical focus of the project was mainly on the introduction of the laboratory approach, the use of historical machines, and the analysis of exploration and proving processes (Antonini & Martignone, 2011).

### Implementation and communication

From 2008, eight different laboratories were set up with several copies of different mathematical machines. A teacher education project was developed dealing with the laboratory approach and the use of mathematical machines in mathematics classrooms (Bartolini Bussi *et al.*, 2011). About 120 teachers were involved in laboratory activities and were placed in learning situations (acting as students) under the guidance of a teacher educator. Later, they were encouraged to analyse the learning processes they experienced in the laboratory activities (acting as professionals). The teacher educators then worked with teachers during the design, development and the subsequent discussion of activities carried out in the teachers' classrooms. The scope of the teacher education project was the same in each of the eight provinces involved: seven half-day meetings over three to four months.

During the teacher education project, the materials used were published on the MMLab website [12]. Teachers also shared materials using the Moodle e-learning platform. The public documentation of the teaching experiments consisted of: the reports of the teaching experiments written by the teachers that carried them out in their classrooms; a PhD thesis (Garuti, 2011); video reports of public events [13]; a number of scientific communications and papers in national and international journals and proceedings [14]; and a final comprehensive report of the first phase of the project (Martignone & Mascherini, 2010). Preparation of a second report is in progress. The published report includes the *voices* of the different participants:

- teacher educators / researchers presented the project's theoretical framework and analyzed the project as a form of teacher education, from the perspective of the international literature;
- the advisors for the laboratories illustrated the local settings;

- the administrators of the teachers' centres set the project within the rich context of the different initiatives carried out locally;
- the tutors of the classroom experiments analyzed the relationships between the teacher education project and its implementation in the classroom;
- the teachers (as either students or professionals) prepared scientific reports about their experiments, focusing on the teaching experiment goals, the methodology adopted, a description of the experience, the analysis and a final reflection on the results; an external observer analyzed the experiment from the perspective of general education;
- policy makers analysed the synergy between this teacher education project and other resources existing in the region.

Most of the materials were written in Italian for local distribution, although some studies have been published in international conference proceedings in other languages.

### Evaluation and impact

In Emilia Romagna, the project led to the creation of a network of laboratories (technical resources) and, above all, a network of groups of teachers (human resources) with expertise in using a laboratory approach and in the didactical use of mathematical machines. In all the provinces, the teachers' engagement continues: they are now teacher educators for locally organized courses, which serve to continue the activities of the project. Some teachers have developed important links between schools, groups of teachers, local centres, universities and other national institutions and they are involved in other education and research projects. The project has also been successfully extended to a neighbouring province outside the region and has attracted attention in other regions all across the country.

Within the whole project, several specific research studies have been conducted. Major research findings concern the study of the transition from tasks for teachers, to tasks for students (Bartolini Bussi *et al.*, 2011; Martignone, 2011) and critique of the literature on mathematical knowledge for teaching (Ball *et al.*, 2008) to argue for the inclusion of a cultural dimension (Garuti & Martignone, 2010; English version in preparation).

### Challenges and further steps

The second phase of the project (2012), which will see its extension to all provinces, is in progress with a new grant that highlights the satisfaction of policy makers. The major challenge now is to construct a regional network to connect the local networks of teachers from every province.

### Discussion

The use of a laboratory approach in mathematics classrooms in Italy has been highly successful, as indicated by research findings, formal evaluations, continued funding and nomi-

nations for awards [15]. In spite of widespread international reporting, however, there has been limited impact on the international research community. To our knowledge, only a few limited classroom experiments have been conducted and analyzed (for a review, see Bartolini Bussi & Maschitto, 2006). In the past, we attributed this fact to the scarcity of large collections of reproduction mathematical instruments. But the situation seems to be more complex: some institutions in different countries did, in fact, borrow or buy sets of mathematical machines [16] and put together successful exhibitions for students, teachers and parents. The instruments were used, however, for the popularization of a cultural approach to mathematics rather than for research in mathematics education.

We have started to wonder what the reasons might be for this (relative) failure, given the success of the MMLab-ER in Italy. Based on our experience so far, it seems to us that the cultural background to the project is a factor in the development of this kind of innovation and related studies. In the case of MMLab-ER, the development of a laboratory approach relied on:

- long-term processes;
- a high level of commitment from teachers;
- institutional constraints, such as national standards and school organization;
- a tradition of focusing on the history of ideas;
- the study of geometry across all the school levels;
- a shared system of beliefs that gives value to students' active exploration, including low-achievers and students with special needs;
- the possibility for teachers to introduce a particular innovative path into their planning.

These are all elements of the cultural background that we found in our context, but that are not likely to be found in every context. For instance, in those contexts where the mathematics curriculum is very prescriptive, it would be more difficult to find teachers willing and able to introduce laboratory activities if such activities are seen as far from the textbook approach. This point applies to all studies involving long-term processes, since, without teachers' involvement, it is not easy at all to design innovative mathematical learning environments.

In this article, we included the section on the cultural background of the project, drawing on our experience of face-to-face communication at international conferences and other similar occasions. A culturally framed project like the one we have sketched has the potential to show how the project exploited international literature on the one hand, and how it might enrich it, on the other hand. The keyword *culture* is a thread running through the whole project. For instance, the choice of a cultural perspective in the curricula developed by the Italian Mathematical Union and exploited by the Ministry of Education as from 2007, the design and analysis of our project for teacher education, and the need to complement the most widespread models of mathematical knowledge for teachers with elements in

which cultural aspects are in the foreground, might all be exploited by an international audience.

There is an additional issue, however. In the past, we too have, in most cases, skipped the description of the cultural background of our research studies. This omission partly depended on the space and time constraints of journals and conference presentations, but also on our lack of awareness of the importance of the cultural dimension. We faced this issue most clearly when we started to collaborate with colleagues from East Asia (for example, Bartolini Bussi *et al.*, 2010). To be able to highlight the cultural background of the project, we had to undergo a demanding process of *centering – decentering – recentering* (Raeithel, 1990). Raeithel described the following three models of relationships between actor and observer in scientific enquiry:

- (a) The *naïve problem solver*, who considers the symbolic structure to be inseparable from the perceived reality;
- (b) The *detached observer*, who represents reality by means of symbolic models;
- (c) The *participant observer*, who develops the split between observing and observed subject into a dialogical relation.

Unfortunately, most international communication in the field of mathematics education seems to belong to the first model (a criticism similar to the one raised by Andrews, 2010). Many comparative studies (*e. g.*, Pegg & Krainer, 2008) seem to belong to the second model, where an apparently “objective” grid tries to capture similarities and differences between curricula, research programs, projects for teacher education and so on. The third model is likely to be adopted by researchers who frame their research studies by means of a cultural dialogue between themselves (the *insiders*) and the international audience (the *outsiders*). This dialogue is useful, first, to insiders (including teachers), as it makes them aware of the implicit educational features that are inherited from local traditions and values: they constitute a strong system of beliefs that may be challenged only by becoming conscious of them and of other possibilities which may exist. It is also useful to outsiders, since it makes explicit which elements of their own cultural background deserve discussion and which findings may be transposed from one context to another.

This issue is common to many educational projects. Several years ago in the ICMI Study Conference held in Maryland on “What is Research in Mathematics Education and What are its Results?”, Ed Silver mentioned the QUASAR project, an ambitious 5-year long design experiment, which stimulated and studied the improvement of mathematics instruction in urban middle schools in the United States. Silver raised the difficulties of discussing and evaluating internationally the findings of local projects in mathematics education. Without explicitly mentioning cultural issues, Silver wrote:

Are all mathematics education research questions able to be considered within the international community? [...] If the research questions cannot be disembedded

from their local contexts, then how is it possible to convey to those who wish to understand the research important background information about the context when it is vital to the research question under consideration? It is possible that, despite their obvious relevance to mathematics education within a particular country, some very important research questions may never be considered more generally within the international community? Is it reasonable? Is it acceptable? What implication does this have for the international community itself and the work it does consider? (Silver, 1994, p. 335)

We hope to prompt debate about these issues at the international level, and suggest some additional questions of our own. How can we convey the cultural dimension in mathematics teaching and learning research? How can we exploit internationally the cultural dimension of a local project?

## Notes

- [1] The project was developed by the authors of this article, together with Michela Maschietto and Rossella Garuti.
- [2] [www.mmlab.unimore.it/on-line/Home.html](http://www.mmlab.unimore.it/on-line/Home.html)
- [3] For photos, descriptions and virtual animations of the machines, see [www.macchinematematiche.org](http://www.macchinematematiche.org)
- [4] The complete volumes, in Italian, can be downloaded at [umi.dm.unibo.it/area\\_download—37.html](http://umi.dm.unibo.it/area_download—37.html)
- [5] The following quotes are from [umi.dm.unibo.it/downloads/icme10.pdf](http://umi.dm.unibo.it/downloads/icme10.pdf)
- [6] [www.education.gouv.fr/bo/2008/hs3/default.htm](http://www.education.gouv.fr/bo/2008/hs3/default.htm)
- [7] See, for instance, [maths.ac-creteil.fr/spip/IMG/pdf/Demarche\\_investigation\\_socle\\_creteil\\_2011.pdf](http://maths.ac-creteil.fr/spip/IMG/pdf/Demarche_investigation_socle_creteil_2011.pdf)
- [8] See the exhibition “Geometry on the fingers” at [www.mmlab.unimore.it/site/home/mostre.html](http://www.mmlab.unimore.it/site/home/mostre.html)
- [9] Up to 1999, no specific methodological program was offered for pre-service secondary mathematics teacher education. From 1999 to 2009 a special 2-year course was added, following university degrees in mathematics or science, to be qualified for secondary school mathematics teaching, but no national evaluation has been done to check whether mathematical laboratory methods have been realized in all the Universities. Now, in 2012, after a three-year suspension of the former program, a new reform is to start.
- [10] [www.mathunion.org/icmi/other-activities/database-project/introduction/italy/](http://www.mathunion.org/icmi/other-activities/database-project/introduction/italy/)
- [11] [archivio.pubblica.istruzione.it/argomenti/gst/documenti.shtml](http://archivio.pubblica.istruzione.it/argomenti/gst/documenti.shtml)
- [12] [www.mmlab.unimore.it/on-line/Home/ProgettoRegionaleEmiliaRomagna.html](http://www.mmlab.unimore.it/on-line/Home/ProgettoRegionaleEmiliaRomagna.html)
- [13] [www.didatticaer.it/progetti\\_regionali/progetto\\_scienze\\_tecnologia.aspx](http://www.didatticaer.it/progetti_regionali/progetto_scienze_tecnologia.aspx)
- [14] [www.mmlab.unimore.it/site/home/progetto-regionale-emilia-romagna/risultati-del-progetto/articoli-di-ricerca-sul-progetto.html](http://www.mmlab.unimore.it/site/home/progetto-regionale-emilia-romagna/risultati-del-progetto/articoli-di-ricerca-sul-progetto.html)
- [15] The project was evaluated as excellent by the international committee of the Altran Foundation for Innovation and selected among the 6 finalists of the Altran award in 2004 (see [www.altran-foundation.org/fileadmin/medias/1.fondation/documents/Altran\\_Foundation\\_-\\_2004\\_Award.pdf](http://www.altran-foundation.org/fileadmin/medias/1.fondation/documents/Altran_Foundation_-_2004_Award.pdf))
- [16] [www.macchinematematiche.org/index.php?option=com\\_content&view=article&id=60&Itemid=128](http://www.macchinematematiche.org/index.php?option=com_content&view=article&id=60&Itemid=128)

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