# IN SEARCH FOR THEORIES: POLYPHONY, POLYSEMY AND SEMIOTIC MEDIATION IN THE MATHEMATICS CLASSROOM

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*This theoretical report addresses the theme of the PME Conference ("In search for theories in Mathematics Education"). The history of two interlaced research programs (Mathematical Discussion and Mathematical Machines) headed by the author is outlined, together with the merging of both, combined with studies on information and communication technologies. They are the roots of the theoretical framework of semiotic mediation after a Vygotskian approach (Bartolini Bussi & Mariotti, 2008). May this framework answer the present needs of focusing cultural historical issues and the teacher's role in the teaching-learning process within the mathematics classroom?*

## INTRODUCTION

This theoretical report reconstructs the scientific and cultural roots of a theoretical framework about the relationship between artifacts and signs in the classroom process after a Vygotskian approach, with emphasis on the teacher's guiding role (Bartolini Bussi & Mariotti, 2008). The narration outlines the development of two research programs (called the *Mathematical Discussion program* and the *Mathematical Machines program*) and offers an annotated bibliography of both. The two programs, headed by the author, were developed independently from each other for years before giving rise to a shared theoretical framework as a result of a dialogue between empirical and theoretical issues. Mariotti is credited for joint elaboration of this framework, mainly (but not only) thanks to her expertise in information and communication technologies (ICT, e. g. Mariotti, 2002). All this was developed within the paradigm of *Research for Innovation in Mathematics Education*, as empirical-theoretical classroom research (Arzarello & Bartolini Bussi, 1998).

## THE MATHEMATICAL DISCUSSION PROGRAM

In the 80s, the author and a group of mathematics teachers (grades 1 - 8) started to study the conditions for realizing effective whole class interaction. Within the European tradition of teaching and learning, we felt uncomfortable with the one-sided focus on learners' activity and on peer interaction that characterized the constructivist approach (dominant in those years in the field of mathematics education). We did not consider this focus respectful of the cultural role of teachers: hence a cultural historical perspective was assumed (Vygotsky, 1978, 1981).

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**Vygotsky: obučenie.** The idea of *obučenie,* i.e. the bilateral process of transmission and appropriation of knowledge, skills and methods, carried out by the teacher and the learner together (Vygotsky, 1990, p. xx), was a warning against reductionist approaches. We were especially interested in the Vygotskian asymmetry between the expert adult and the young children in the *zone of proximal development* and in the process of *internalization* (Bartolini Bussi, 1998a).

**Leontev: activity, actions, operations.** We focused long term processes, which last weeks, or even months. This depends, first, on the organization of the Italian school system (where a teacher teaches the same classroom for three or even five years) and, second, on the belief that many relevant changes can be observed only in the long run. Leontev activity theory (1978) offered the distinction between the three levels of *activity* (collective and directed towards an object-motive), *actions* (goal direct processes) and *operations* (the way of carrying out actions in variable concrete circumstances, Bartolini Bussi, 1996, p. 15ff). Because of the focus on the teacher's role, actions and operations were studied mainly with reference to this acting subject. As it is impossible to design a priori the complexity of interactional processes, the study of teacher's on the spot improvisation was needed.

**The Italian comedy of art: anticipated improvisation.** To approach the issue of teacher's operations, we considered the idea of *improvisation*, as emergent in the Italian comedy of art (Fo, 1987). Actors do not always invent cues; rather they often choose them in a repertory that has been studied diligently according to the different situations which may occur. Hence, our effort was directed towards eliciting goals of the teacher from specific classroom situations and towards collecting "constellations" of communicative strategies (cues) which had shown effective empirically in fulfilling the goal, in order to construct a repertory to be learnt (Bartolini Bussi, 1998b). A recent and more complete work has been carried out by Falcade (2006).

**Bachtin: polyphony.** The metaphor of *polyphony* (Bachtin, 1968) was adopted to consider the system of utterances produced by students and teachers or by evoked authors of texts (e.g. historical sources, textbooks). We used the word *voice* after Bachtin to mean a form of speaking and thinking, which represents the perspective of an individual, i.e. his/her conceptual horizon, his/her intention and his/her view of the world (Bartolini Bussi, 1996).

**The construct of Mathematical Discussion orchestrated by the teacher.** After some years of empirical work in the mathematics classrooms from grades 1 to grade 8, we described in a precise way the specific form of classroom interaction we were working on, i. e. the *Mathematical Discussion orchestrated by the teacher.*

> The Mathematical Discussion [orchestrated by the teacher] is polyphony of articulated voices on a mathematical object (e.g. a concept, a problem, a procedure, a structure, an idea or a belief about mathematics), that is one of the motives of the teaching-learning activity (Bartolini Bussi, 1996, p. 16).

The recourse to musical metaphors was not accidental. Besides borrowing the words polyphony and orchestration from Bachtin we wished to emphasize also the importance of imitating voices in counterpoint. This position was very strongly influenced by Vygotsky's emphasis on *intellectual imitation* as one of the basic paths of cultural development of the child (Vygotsky, 1978). In other words, we stated very firmly that imitation is essential in the teaching-learning process and not opposed to creative thinking (for a contrast with constructivist perspectives on mathematical discussion see Bartolini Bussi, 1998a, p. 14 ff.).

Several experiments were carried out in the following years in grade 1-8 classrooms (Bartolini Bussi, 2007; Bartolini Bussi & Boni, 2003; Bartolini Bussi et al., 1999, 2005, 2007). In parallel, Mariotti implemented systematically mathematical discussions in ICT environments (e. g. Mariotti, 2002; Cerulli, 2004).

## THE MATHEMATICAL MACHINES PROGRAM

Most of the research studies quoted at the end of the previous section concern concrete artifacts taken from the history. This is consistent with one of the major tenets of cultural historical school. Actually Vygotsky did not study only language but also the role of artifacts in the cognitive development (Bartolini Bussi & Mariotti, 2008, p. 751) and suggested a list of possible examples:

> various systems for counting; mnemonic techniques; algebraic symbol systems; works of art; writing; schemes, diagrams, maps, and mechanical drawings; all sorts of conventional signs, etc" (Vygotsky 1981, p. 137).

In the Laboratory of Mathematical Machines (www.mmlab.unimore.it) headed by the author, more than 200 artifacts have been reconstructed drawing on the historical phenomenology of geometry, from the classical age to the 20th century. A *mathematical machine is a tool that forces a point to follow a trajectory or to be transformed according to a given law.* The most common mathematical machines are the pair of compasses (that forces the point with the graphite lead to draw a circle). Since the 80s empirical classroom activity was carried out at high school level (grades 9 - 13) by the members of the Laboratory team. In all the experiments, small group work with a mathematical machine was realized before whole class discussion of findings. The historic epistemological analysis was in the foreground whilst the study of classroom organization and processes came later (Bartolini Bussi & Pergola, 1996; Bartolini Bussi, 2005; Bartolini Bussi & Maschietto, 2006).

**Rabardel: artifacts and instruments.** Rabardel's instrumental approach (1995) is based on the distinction between artifact and instrument: the artifact is the material or symbolic object per se whilst the instrument is a mixed entity made up of both artifact-type components and schematic components (utilization schemes). The utilization schemes are progressively elaborated by the user during artifact use to solve a task; thus the instrument is a construction of an individual, it has a psychological character and it is strictly related to the context (Bartolini Bussi &

Mariotti, 2008, p. 748 ff.). The elaboration and evolution of the instruments (*instrumental genesis)* can be articulated into two processes: *instrumentalisation*, concerning the emergence and the evolution of the different components of the artifact, e.g. the progressive recognition of its potentialities and constraints; *instrumentation*, concerning the emergence and development of the utilization schemes. The two processes are outward and inward oriented, respectively from the subject to the artifact and vice versa, and constitute the two inseparable parts of instrumental genesis. Both analyses have been applied to mathematical machines (Bartolini Bussi & Maschietto, 2006, ch. 4; Martignone & Antonini, in press).

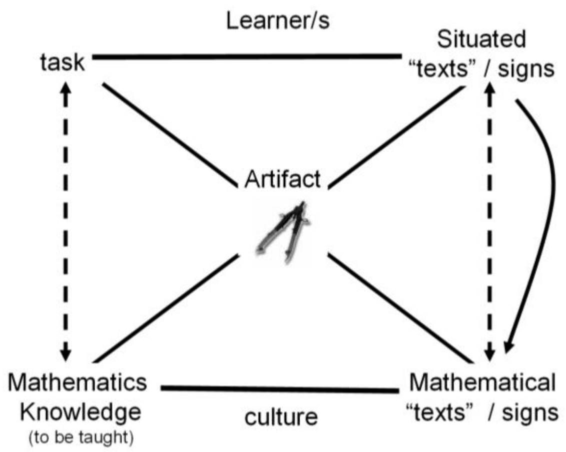
**Wartofsky: polysemy.** Wartofsky (1979) analyses artifacts from epistemological perspective. According to him, the term artifact has to be meant in a broad sense (Bartolini Bussi & Mariotti, 2008, p. 760 ff.), including tools (*primary artifacts*), representations (*secondary artifacts*) and theories (*tertiary artifacts*). Consider the case of the pair of compasses. One may use it to draw round shapes (primary artifact), to find a point at a given distance from two given points, according to Euclid's definition of circle (secondary artifact) or to evoke the Euclidean geometry (tertiary artifact). The introduction of an artifact in a classroom does not automatically determine the way it is used and conceived of by the students (i. e. *polysemy* emerges) and may create the condition for generating the production of different voices. For each artifact, one may analyse a priori the *semiotic potential* that links the meanings emerging from its use, aimed to accomplish a task, and the mathematical meanings evoked by that use. This analysis suggests ways of starting, monitoring and managing *polyphony* in classroom interaction.

Wartofsky's analysis offered an epistemological perspective, whilst Rabardel's instrumental approach offered a cognitive perspective on the use of mathematical machines.

## SEMIOTIC MEDIATION: THE MERGING OF TWO RESEARCH PROGRAMS

In the early years the didactical analysis in the field of experience of Mathematical machines looked still weak, whilst in the Mathematical Discussion program it was stronger. The merging of two programs aimed at deepening the construct of *semiotic mediation*, as conceived by Vygotsky (1978, p. 39-40). Polyphony caught the linguistic, whilst polysemy caught the instrumental aspects. Moreover the presence of concrete artifacts emphasized the importance of other semiotic systems in addition to language (e. g. gestures, drawings; Maschietto & Bartolini Bussi, 2009).

It is beyond the aim of this short report to present the resulting theoretical framework (Bartolini Bussi & Mariotti, 2008), that encompasses all the issues above and includes also ICT. Two schemes may recall and outline the framework. The first (fig. 1) represents the system of semiotic activity that hints at the teacher's roles; the second (fig. 2) represents the didactical cycle that hints at long term processes.



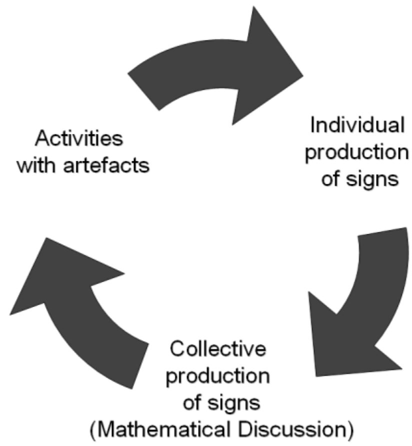




Figure 1: Artifacts in semiotic mediation. Figure 2: The didactical cycle.

The fig. 1 represents what happens when a student (or a small group of students) is given a task, that, according to teacher's intention, is related to both a piece of the mathematics knowledge to be taught and the use of an artifact (e. g. a pair of compasses). A solution of the task, although correct, might be only a technical solution (where the artifact is used as primary one), with situated "texts" (signs) and without any awareness of mathematical meaning. On the right side of the diagram, it is represented the teacher's aim to transform the above situated "texts" (signs) into mathematical "texts" (signs) which might be easily related to the piece of mathematical knowledge to be taught. The teacher's main roles are the following: (left) to construct suitable tasks; (right) to create the condition for polyphony, eliciting the polysemous feature of the artifact and to guide the transformation of situated "texts" (signs) into mathematical "texts". In this way the teacher mediates mathematical meanings, using the artifact as a tool of semiotic mediation. Without teacher's intervention, there might be a fracture between learner/s (top) and culture (bottom) planes, hence no learner's construction of mathematical meanings.

This is not a short term process. The fig. 2 shows the articulation of classroom processes over time, from individual or small group solution by means of the artifact, together with the individual production of "texts" (signs), to collective production of "texts" (signs) in mathematical discussion orchestrated by the teacher (Bartolini Bussi & Mariotti, 2008). Fine grain analysis of the teacher's role in this long term process has been carried out by Falcade (2006) and is still in progress in specific situations.

As motives of activity include "existing" mathematical meanings (often expressed in a crystallized form), a critical reader might object that such a teleological approach contrasts Bachtin's idea of polyphony. On the contrary, we claim that, according to the Vygotskian construct of internalization (1978, p 56), what is internalized by students is not the crystallized result but rather the interpersonal process of construction, that is truly polyphonic. There are always traces of polyphony in students' protocols, which may involve other semiotic systems, e. g. gestures (Maschietto & Bartolini Bussi, 2009). As authors end novels, teachers lead students to construct/appropriate 'existing' mathematical meanings: it is not a finish but rather a new start. As Vianna & Stetsenko (2006) claim, from a Vygotskian perspective:

> Present generations never invent their world and themselves from scratch but inevitably continue their past, even if by completely breaking away from it. However, it is also clear that the past does not simply evolve in the present but is enacted by each generation of people each time anew and in view of the present and the future (which is flexible too), through innovative and bold contributions to it. This mutual interpenetration of past and present can be well captured by the metaphor that the present without the past is blind, but the past without the present is powerless (p. 100-1).

## CONCLUDING REMARKS

The main contributions of this comprehensive framework concern the role of history and culture and of tools (artifacts from both history and ICT) as mediators under the guide of the teacher, and the study of teacher's asymmetric role (as a guide) in the mathematics classroom.

When the Mathematical Discussion program was started, it was not easy to find interlocutors in the scientific community, because the focus on teaching was mistaken for neglecting learning. After more than twenty years the situation is expected to be different. According to Sfard (2005), in the 21st century, from the era of the learner we have entered the era of the teacher. Yet, is it real? Vianna & Stetsenko (2006) have carefully analysed some research programs from US and UK and have found a pivotal difference in views on the role of history and culture and of teaching in development, in comparison with Vygotskian tenets. This difference seems to draw on deep roots. As Clarke (2007) observed

> Whether we look to the Japanese “gakushushido", the Dutch “leren” or the Russian “obučenie”, we find that some communities have acknowledged the interdependence of instruction and learning by encompassing both activities within the one process and, most significantly, within the one word. In English, we seem compelled to dichotomise classroom practice into Teaching or Learning (p. 23).

Yet, now the world of mathematics education communities is expanding and attitude is changing: *may the theoretical framework of semiotic mediation after a Vygotskian approach, as developed by Bartolini Bussi & Mariotti* (2008), *be assumed as a useful theory for research on teaching - learning in the mathematics classroom?*

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