Intercultural dialogue and the geography and history of thought

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First vignette: China and Italy

Mariolina (Bartolini Bussi) is talking at a conference with a Chinese colleague, Xuhua. She has just presented a report on fractions and is writing on the whiteboard with a felt pen. Suddenly Mariolina notices that X writes fractions in a strange order, first the denominator, then the fraction bar and eventually the numerator:

Mariolina: Why do you write fractions in this way?
Xuhua What do you mean? How should I write them?
Mariolina: I mean the order. We write them in the reverse order (top-down): first the numerator then the fraction bar and last the denominator.
Xuhua Very strange, indeed! How do you know how many pieces you wish, if you do not know in how many pieces you have cut the whole?

Second vignette: Italy and Burma (Myanmar)

Mariolina and Alessandro (Ramploud) are talking with two Burmese colleagues (Thein Lwin, a mathematician, and Ko Ta, a doctor and coordinator of a network of Monastic schools) who are visiting our department:
Mariolina: How do you write fractions in Burmese? For instance two thirds
Thein Twin: [*is a bit surprised, writes 2/3 top-down*] Why?
Mariolina: I have read in Wikipedia that the Burmese order is the same as the Chinese one: bottom-up.
Thein Lwin: [*shakes his head*] No, it's the same as yours!
Ko Ta: [*smiles*] I am not a mathematician!
Ko Ta closes his eyes, takes a pencil and traces gestures in the air. Alessandro has the impression that Ko Ta is looking for a kind of motion memory of the gesture used when he was a kid in a primary school. After some seconds, Ko Ta smiles and shows a bottom-up process: first 3, then the fraction bar and eventually 2.
Thein Lwin: [*smiles and nods*] He's right. I agree!

These two vignettes tell us a simple story. Chinese and Burmese are in the same family of Sino-Tibetan languages. Hence, it is not surprising that their way of saying fractions (and the process of writing fractions) are similar. Yet in Chinese the traditional process of writing (order) and saying fractions is still the same as in the past, taught in the same way in textbooks, whilst in Burmese it seems that a "Western" habit is changing the tradition. It would be interesting to know whether this process depends on the effect of colonialism (that for decades designed the Burmese education system according to the British tradition) or on the effort to run after Western mathematics and mathematics education as a way to overcome the negative effects of military rule. This issue deserves further analysis; however, it helped the participants in the interaction to reflect on each other's own *un-thought*. Here we are quoting Jullien (2006), the French philosopher and sinologist, who explains his decision to start to study Chinese and to move to Beijing as a way to understand better the European and Greek philosophy. To observe one's own culture from a distance helps to understand one's own un-thought. The *geography of thought* (Nisbett, 2003) allows us to become aware that our beliefs are relative and that they could have been different had we come from different parts of the world (Bartolini Bussi & Martignone, 2013; Bartolini Bussi et al, 2013).

The history of thought

These stories also raise our curiosity to learn about the *history of thought*. All European languages now share the top-down writing process of fractions and the consequent naming order. What are the roots of this process? *Liber Abaci* (by Leonardo Fibonacci, who introduced the so-called Indo-Arabic notation to Europe), reads:

> When above any number a line is drawn, and above that is written any other number, the superior number stands for the part or parts of the inferior number; the inferior is called the "denominatus" (denominator), the superior the "denominans" (numerator). Thus, if above the 2 a line is drawn, and above that unity [1] is written, this unity stands for one part of two parts of an integer, *i.e.* for a half, thus ½. (As quoted in Cajori, 1928, p. 269)

Hence we know that the order of describing fractions (and probably, we assume, also that of writing fractions) for Leonardo Fibonacci was (in line with the "Eastern" order):

denominator → fraction bar → numerator.

Probably the reverse top-down order used later was an effect of the standard way of writing from the top to the bottom of the sheet. The final written products are the same!

Yet there is still the issue of ordinal numbers. Why is the denominator expressed in ordinal numbers? This is even more counter-intuitive. We have not yet found any satisfactory answer to this second question in the books on the history of mathematics or in conversations with historians. We guess that it is related to the importance (as it was already in ancient Egypt) of unit fractions that were used more often than other fractions, and, in some cases, instead of other fractions. There were rules (also studied by Leonardo Fibonacci) that allowed the writing of any fraction as the sum of unitary fractions and this writing helped to solve practical problems in a very effective way. For instance, to divide 5 pizzas among 8 children, one can say that each child has 5/8 of a pizza, but this requires cutting each pizza into 8 pieces and giving 5 pieces to each child. It is quite different from what somebody would do in everyday life! The sum

$$\frac{5}{8}=\frac{1}{2}+\frac{1}{8}$$

mirrors the more natural idea of cutting 4 pizzas in half (to give one half to each child) and then dividing the last one into 8 parts, to give a small piece more to each child. This solution is similar to the one found in ancient civilizations and in the Liber Abaci itself. The recourse to the sequence of unitary fractions in problem solving could have been so natural and frequent that they were considered a special genre of numbers, similar to the natural ones:

$$1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\frac{1}{6} \text{ and so on.}$$

and so on. In this sequence, the order corresponds to the wording of the denominators (at least from the third one). We know that the systematic approach to general fractions with any numerator is a recent idea. Even more recent is the idea of considering fractions in mathematics education as numbers to be represented in a number line, exactly like the whole numbers.

Implications for mathematics education

The case of fractions is just one example of the richness of taking a different perspective on our own un-thought about a mathematical process. Discovering that some issues that had been considered obvious are, on the contrary, the products of long and complex cultural processes prompts teachers to reflect on their beliefs and on the hidden choices made in their context. Although a direct transposition might be impossible, we know that Western languages and traditions are not always the best ones to hint at the genesis of some mathematical processes. In the case of fractions, some Eastern languages seem to be to be facilitators for the construction of meanings (see Siegler *et al.*, 2013).

> **Third vignette: Italy—interaction between an expert and a low achiever**
> Anna (Baccaglini-Frank) is working with a low achiever, L, using the software *Motion Math* [1] an app for the iPad, in which learners have to tilt the device to make a falling ball containing a fraction fall towards the right point on the number line [0,1] (for a video, see [2]).
>
> L seems to be confused by the task. Without an intuition about the position of the fraction on the line it is not easy at all to tilt the device quickly enough during the very short falling time.
> Anna tries to help him by reading the falling fraction. She is using the Western mode: two thirds, three fourths, and so on.
> Anna: [*suddenly changes the way of reading*] Let's name the fractions as Chinese do!
> Anna: [*½ falls*] Of two parts, take one!
> Anna: [*¾ falls*] Four parts, three!
> L is a bit surprised, starts to be less anxious and improves very quickly his performance. The improvement is more evident with unitary fractions (e.g. 1/5).
> L: Oh yeah, I have to divide the segment into 5!

The same happens with other low achievers.

Motion Math exploits both epistemological and cognitive analyses of fractions (Riconscente, 2013), emphasizing, on the one hand, the importance of using the number line to give coherence to the study of fractions and of whole numbers and, on the other hand, the neurological evidence of the mental number line (Zorzi *et al.*, 2002). Moreover, *Motion Math* exploits embodied learning and, in particular, the integrated perceptual-motor approach (Nemirovsky *et al.*, 2012) in the development of such a mental number line. From her research on students with mathematics learning difficulties (Karagiannakis et al., in press), and in particular when engaging in interventions with low achievers, Anna is learning to combine neuroscientific findings with the outcomes of the intercultural semiotic analysis discussed in our research group, to smooth the scarce transparency of the Italian wording.

This very short episode from a study in progress shows the synergy between intercultural dialogue, neuroscience and technology for defining effective teaching-learning situations. We hope that this synergy will be further and more deeply developed in the future, and applied in mathematics teacher education and development.

Notes

[1] motionmathgames.com

[2] www.youtube.com/watch?v=hmm0D90vcYI

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