Research in didactics of mathematics

Emerging themes

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The French didactic tradition in mathematics

- ARDM (**A**ssociation for **R**esearch in **D**idactic of **M**athematics ardm.eu): a member of CFEM
- Presentation of the French didactic tradition at ICME13 (Artigue et al. 2019)
- Importance of the theories
- Strong links with mathematics and mathematicians
- Design of teaching and learning environments
- Empirical research
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Since ICME13, new developments in France

- Formative and summative **assessment**
- within national and european projects
- models for a didactical study of assessment

(e.g. Grapin 2015, Sayac 2018, Coppé 2018)

Since ICME13, new developments in France

- **Algorithmics**
- A new field in France in the recent curriculum reforms from primary to upper secondary school
- New research about this evolution, its consequences, and more generally about the links between mathematics and computer science

(e.g. Durand-Guerrier, Meyer & Modeste 2019, Laval 2018)

Since ICME13, new developments in France

- **Practices of mathematicians**
- Studies of research practices of mathematicians and of researchers in other linked sciences (such as physics, life science, computer science)
- (e.g. El Hage & Ouvrier-Buffet 2018, Grenier-Boley 2019, Yvain 2018).

Since ICME13, new developments in France

- **Students with special needs**
- A focus on the mathematical knowledge as a central content in the educational project of cognitive-disabled students (e.g. Assude et al 2014).
- Students with special needs (mathematics learning disabilities, dyspraxia disabilities) and remedial interventions in the classroom (e.g. Peeters & Ouvrier-Buffet 2019, Petitfour 2018).

Since ICME13, new developments in France

- **Structuration of theoretical backgrounds**
- Example of the MWS (Mathematical Working Space) of analysis (Montoya Delgadillo & Vivier, 2016), at the university level (focusing on the notions of convergence, optimization, modeling, ...) with international collaborations (Chile, Germany and Mexico for instance)
- Focus on engineering students (Gaona's PhD, 2018)

Next presentations

Hussein Sabra

Teachers' collective documentation work

Julia Pilet

Collaborations between researchers and teachers

Nicolas Grenier-Boley

Higher-education

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Conclusions

1.The study of Teachers' Collective Documentation Work

The case of the reflective investigation methodology

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Context and aims of the study

- Sésamath an online association of mathematics teachers :
  - To design resources with/for teachers.
  - About one million visits each month on its website.
- A Sésamath project : designing e-textbook for grade 10
- The interaction between individual resources and shared resources, and its impact on the professional knowledge shared within the community (Sabra & Trouche, 2011).
- The Documentational Approach to Didactics (Gueudet, Pepin, & Trouche, 2012).

The reflective investigation methodology

- Five principles grounding the reflective investigation methodology (Trouche, Gueudet, & Pepin, 2018):
  - The *broad collection of the material resources;*
  - The *long-term follow-up;*
  - The *in- and out-of-class follow-up*.
  - The *reflective follow-up* of the documentation work ;
  - The principle of *confronting the teachers' views on her documentation work.*
- The data collection tools depends on the teachers' collective work, and on the research issues.

The reflective investigation in the case of Sésamath e-textbook project

- Four years follow-up, Several types of data were collected:
  - The web-based *discussions strings*; and the resources platform.
  - The *resources* suggested by the members on the mailing list and platform.
  - The resources designed during the collective documentation work.
  - The *schematic representations of the collective documentation work*
  - Several members for filling out a logbook (kind of “event log”)
- The individual documentation work of some members was followed on the platform but also in the classroom.

The reflective investigation and data analysis

- What the researcher is asking for and for which purpose :
  - to clarify the purpose of the research;
  - the necessity to consider the *methodological contract* (Sabra, 2016)
- The reconstruction of ongoing processes
  - Combining professional knowledge expressed and the resources shared and designed
  - Evidencing (or not) the existence of shared documents

2. A long-term collaboration between researchers and middle school teachers on algebra

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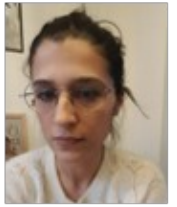
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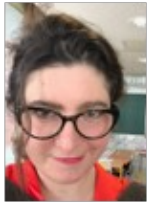
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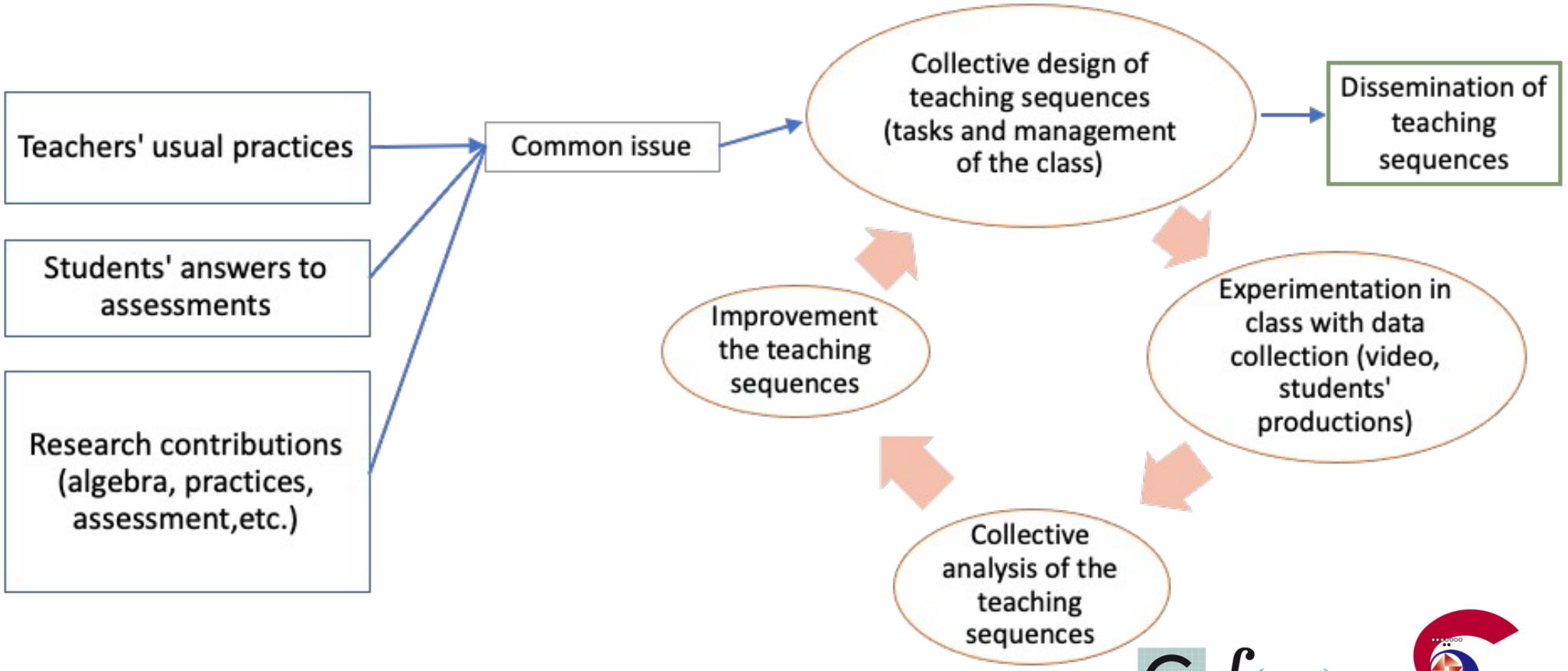
Stakes of this collaboration

- Observations in France
  - A teaching profession that is often solitary, with a great variability of practices among the teachers (Talis, 2013, 2018, Praesco, 2021)
  - A desire to have teachers work together, following the Lesson Study (Mission Villani-Torossian, 2018)
  - Fragility of students' mathematical skills at the end of middle school (15 years old), especially in algebra (CEDRE, 2020)
- Goals
  - Long-term professional development for teachers
  - Design of resources for the teaching of algebra
  - Production of research results (teaching/learning, assessment, teacher education)

Specificities of this collaboration

- Integration into institutionally recognized programs
  - LéA (Associated Education Place) attached to IFE (2014-2020)
    “LéA Roger-Martin-du-Gard “
  - Group of the IREM of Paris (Institute of Research on Mathematics Education) (since 2016)
    Group “ Assessment practices in numerical and literal calculus “
- Iterative approach close to other collaborative frameworks like Lesson Study (Hart et al., 2011 ; Dudley 2014), Collaborative Research (Bednarz, 2013), Cooperative Engineering Research (Sensevy et al., 2013), etc.

Iterative approach to design teaching sequences supported by research



Research dissemination and impact

- Positive long-term changes in teaching practices and students' skills (Pilet, Allard, Horoks, 2019 ; Pilet et Horoks, 2018 ; Horoks et Pilet, 2018 ; Grugeon-Allys et al., 2019)
- A significant diffusion extended to the teachers
  - Availability of our resources via the IREM of Paris
  - Setting up of a continuous teacher education program each year, which targets all the middle-schools of one or two areas around Paris (Académie de Créteil), at the request of the school institution (since 2017)
  - Facilitation of workshops or presentations (Lea days at the IFE, etc.)

3. Research in didactics of mathematics: the case of higher-education

Two examples

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Teachers-researchers' teaching practices (1)

- Many international research works related to this issue:
  - pedagogy of TRs' teaching practices (Annoot & Fave-Bonnet, 2004 ; Berthiaume, 2007)
  - a need to address this issue through an approach based on their discipline (Becher, 1994 ; Neumann, 2001, Poteaux, 2013)
  - a need to study the influence of research practices on Trs' practices (Biza, Giraldo, Hochmuth, Khakbaz & Rasmussen, 2016, ICME13)
- A major question: what is the discipline's imprint on TRs' teaching practices at the beginning of university?

Teachers-researchers' teaching practices (2)

- A general research about ideal or declared practices:
  - involving four disciplines (Chemistry, Geography, Mathematics, Physics), TRs from several universities
  - based on semi-directive individual interviews
- An example of result: identification of similarities between disciplines, and of contrasts that might be interpreted as epistemological differences
- Perspectives: need to study *in situ practices* (Bridoux, de Hosson & Nihoul, INDRUM2020), to deepen the relationship between teaching and research from a theoretical point of view
- A book in progress involving several research disciplines (didactics, educational sciences, sociology)

Klein's double discontinuity

- Many research between secondary education and postsecondary education (Klein's first discontinuity): Gueudet, (2008), Gueudet, Bosch, diSessa, Kwon & Verschaffel (2016, ICME13)
- A need to address the transition between postsecondary education and school mathematics for (future) teachers (Klein's second discontinuity): Winsløw & Grønbæk (2013), Gueudet et al. (2016)
- Two perspectives: 1) to highlight for students the links between university and school mathematics, 2) to provide effective tools to future or current teachers for their didactic work.
- An online international seminar dedicated to these issues (Durand-Guerrier, Grenier-Boley) related to existing networks (DEMIPS, INDRUM)

Conclusions

The vitality of research in didactics of mathematics in France

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New themes presented in the videos

- Teachers' collaboration, digital resources and the documentational approach (Hussein Sabra)
- Teachers and researchers working together in a LéA (Julia Pilet)
- University Mathematics Education: University teachers' practices, Klein's double discontinuity (Nicolas Grenier-Boley)

Stability of the French tradition, and evolutions

- Importance of the theories, grounding the research design – including new theories, like the documentational approach or the Mathematical Working Spaces
- Collaborations with mathematicians, in particular about University mathematics education
- Strong links with the IREMs, associating research, teacher professional development and resources design.

The French tradition as an international tradition

- The French ‘doctoral schools’ welcome PhD students from all over the world;
- Members from more than 20 countries in ARDM ;
- International impact of French theories

  e.g. the theory of conceptual fields. *Tribute to Gérard Vergnaud, who died in June 2021, leaving a major international contribution to mathematics education…*

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Example 2

LéA Roger Martin-du-Gard : http://ife.ens-lyon.fr/lea/le-reseau/anciens-lea/lea-reseau-de-colleges-martin-du-gard

Groupe IREM de Paris : https://irem.u-paris.fr/pratiques-devaluation-en-calcul-numerique-et-litteral

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Example 3

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