National Presentation of Mathematics Education in France at ICME-14

produced by CFEM
French commission on mathematical instruction
commission française pour l'enseignement
des mathématiques

Contents

I Introduction: CFEM

As indicated, this document has been prepared by the CFEM, the French Commission for Mathematical Instruction. It is useful, before entering into the presentation of mathematics teaching in France, to begin with an introduction to the role of this structure and its environment.

1. What is the CFEM?

It is the French sub-commission of the ICMI (International Commission for Mathematical Instruction, CIEM in French). In France, it is a common platform for all professional associations and learned societies related to mathematics and its teaching (ADIREM, APMEP, ARDM, CNFM, *femmes & mathématiques*, SFdS, SMAI, SMF and UPS[1], and also for the Institut Henri Poincaré (IHP) and since 2020 MATh.en.JEANS. It works in partnership with the mathematics group of the General Inspectorate of National Education, now the General Inspectorate of Education, Sport and Research (IGÉSR) and the Academy of Sciences. The role of the CFEM is to encourage cooperation between the various actors of mathematics and its teaching, and their commitment in common actions, to promote a living teaching of mathematics. It acts in this sense towards public organizations and, more generally, towards the society.

Its statutes is that of an association composed only of individual members delegated by the above-mentioned organizations, and of a limited number of co-opted members, which allows for example to associate other associations such as Animath, whose goal is to promote the taste and the practice of mathematics among young people.

2. An international vocation

Through its link with ICMI, the CFEM plays an essential role of international interface. In particular, it coordinates the French participation in the International Congresses on Mathematical Education (ICME) which take place every 4 years: the last two were ICME-12 (Seoul, Korea, 2012) and ICME-13 (Hamburg, Germany, July 2016). This document is written in anticipation of the ICME-14 congress in Shanghai (China) in July 2021 (planned in 2020 it has been postponed by one year due to the Covid pandemic). Note that Luc Trouche, president of the CFEM from 2012 to 2016, is a member of its international program committee.

[1]Assembly of Directors of Research Institutes on the Teaching of Mathematics (ADIREM), Association of Teachers of Mathematics in Public Education (APMEP), Association for Research in Didactics of Mathematics (ARDM), Women and Mathematics, French Society of Statistics (SFdS), Society of Applied and Industrial Mathematics (SMAI), Mathematical Society of France (SMF), Union of Teachers of Scientific Preparatory Classes (UPS).

CFEM also contributes to the international ICMI Studies conducted regularly by ICMI; recently: *Proof and Proving in Mathematics Education* (19), *Educational Interfaces between Mathematics and Industry* (20), *Mathematics Education and Language Diversity* (21), *Task Design* (22) and *Primary Mathematics Study on Whole Numbers*(23), *School Mathematics Curriculum Reforms: Challenges, Changes and Opportunities* (24) and the ongoing *Teachers of mathematics working and learning in collaborative groups* (25). The activity of the CFEM is also stimulated by the international recognition of the research carried out in France on mathematics education, as shown by the distinctions awarded by the ICMI: the Felix Klein medal to Guy Brousseau in 2003 and to Michèle Artigue in 2013; the Hans Freudenthal medal to Yves Chevallard in 2009. Let us also recall the Paul Erdös prize awarded in 2004 to André Deledicq, co-founder of the Kangourou des Mathématiques, and at the initiative of the creation in 1994 of the current CFEM (see the elements for the history of the CFEM by Jean-Pierre Kahane http://www.cfem.asso.fr/cfem/elements-pour-l-histoire-de-la-cfem).

Let us note that Jean-Pierre Kahane (president of ICMI from 1983 to 1990), who passed away in June 2017, and Michèle Artigue (president from 2007 to 2010), who is still very active in ICMI, and also for this ICME-14 congress, have played an important role within ICMI and have allowed the CFEM to benefit from their expertise and their exceptional investment. The arrival in 2021 of Jean-Luc Dorier, President of CFEM from 2002 to 2006, as Secretary General of ICMI in January 2021 is a new element of stimulation.

3. A commitment to the French-speaking world

In July 2000, the World Year of Mathematics, CFEM organized the first Francophone Congress on Mathematics Education in Grenoble. The success of this undertaking created a dynamic that led to the organization every three years of a colloquium called *Espace Mathématique Francophone* (EMF), recognized as an ICMI Regional Conference, to which CFEM continues to contribute actively. Six countries have already successively hosted this important event: France, Tunisia, Canada (Quebec), Senegal, Switzerland and Algeria (EMF2015 took place in Algiers) and again France for EMF2018 (in Paris (Gennevilliers), CFEM was a member of the organizing committee https://emf2018.sciencesconf.org/). The next EMF was planned in Cotonou in Benin in December 2021, it has also been postponed[2].

In addition, the CFEM has supported since its creation the CANP (Capacity & Networking Project) program of ICMI, whose first realization took place in Bamako (Mali)

[2]EMF website (Espace Mathématique Francophone): http://emf.unige.ch/

in October 2011 (the report "Teacher Training in Francophone Sub-Saharan Africa; Five Case Studies: Burkina Faso, Côte D'Ivoire, Mali, Niger, Senegal, in French and English, available on the ICMI website[3]).

4. Some considerations on its national role

The CFEM is the embodiment of the great cohesion of the scientific community related to mathematics education. Its effective role and its actions remain, however, very much constrained by the decision-making system of educational policy in France, which is itself completely linked to the political system: the rhythm of elections, the choice of ministers and the attribution of competences (for example, in 2021, there is a Ministry of National Education, Youth and Sports, and a Ministry of Higher Education, Research and Innovation; there was a single Ministry in charge of education from April 2014 to May 2017). The succession of ministers in charge of national education leads to a succession of decisions directly concerning the teaching of mathematics at the primary and secondary levels, taken in a more or less concerted manner with the mathematical community, and therefore more or less consensual and more or less supported by the latter. As far as their implementation is concerned, both for primary and secondary education, it is based on a central administration (DGESCO), *Direction Générale de l'Enseignement Scolaire* (Department of School Education), and also on a territorial division into *académies*. The implementation at the local level can be quite varied, as can be the relations with the community - the IREMs (*Instituts de Recherche sur l'Enseignement des Mathématiques*, Research Institutes on Mathematics Education), established in each *académie*, can testify to this diversity.

An example is that of the *Stratégie Mathématiques* [137], which is mentioned in the next chapter on recent curricular evolutions. The CFEM participated in its elaboration and was a member of the follow-up committee. This made it possible to have regular and privileged relations with a certain number of institutional actors, including DGESCO, and in return strengthened the collaboration between the components of the CFEM. It is obvious that being considered as an interlocutor facilitates the circulation of information and requires, and therefore favors, this collaboration. On the other hand, the CFEM was simply consulted by the Villani-Torossian Commission [145], in the same way as all its component associations, and, concerning the last reform of the *lycée*, it was consulted by the CSP (*Conseil supérieur des programmes*, Higher Council for Programs), but not directly by the *Comité de suivi de la réforme du baccalauréat 2020* (Baccalaureate Reform Monitoring Committee). Nevertheless, in each consultation, the CFEM expressed a position and made proposals prepared collectively and when an association of the CFEM was solicited on a subject of common interest, it informed the other components.

These considerations are not intended to minimize the role that CFEM plays and can play. The existence of a structure that encourages discussion between components that have essentially distinct objectives, at least in part, is a great strength that

[3] `https://www.mathunion.org/icmi/activities/developing-countries-support/capacity-networking-project-canp`

other school disciplines do not possess. In a period of very rapidly implemented reforms, the organization of the community makes it possible to carry out investigations, analyses, to reflect in depth, to point out dysfunctions (one can cite, for example, the very gendered choices of specialties made by students in the final cycle of the general high school, and also the problems posed by the common core and science teaching), to propose improvements (as with the system of pre-professionalization educational assistants http://www.cfem.asso.fr/actualites/communique-de-la-cfem-et-de-ses-partenaires-sur-les-aed-avril-2021). Even if it is not always heard, our community continues to mobilize so that the teaching of mathematics is of high quality for all students because it believes that it is an essential element in the training of both future citizens and future scientists.

5. Some examples of national actions

The examples that are developed are chosen to illustrate the diversity of actions that CFEM tries to launch or support, with the institution or independently. The possibility of launching actions remains constrained by the structure of the association which, by its status since its creation, has very limited means of financing, and by the difficulty of obtaining subsidies from the ministry, or at least the impossibility of obtaining them with a minimum of anticipation. On the other hand, it should be emphasized that these actions have been carried out thanks to the voluntary investment of many teachers at all levels.

5.1. Forum of Living Mathematics. Launched as part of the *Stratégie Mathématiques*, the *Forum des Mathématiques Vivantes* (Forum of Living Mathematics), orga-

FIGURE 1. Poster of the 1st *Forum Mathématiques Vivantes* 2015. Brochure Panorama of the theme *Mathématiques et langages* of *Forum Mathématiques Vivantes* 2017

nized in conjunction with the *Semaine des mathématiques* (Mathematics Week), offered

training activities for teachers and also a variety of activities for the general public in a few emblematic locations: Paris, Lyon and Marseille for the first one, which was organized by Michèle Artigue, then president of CS-IREM (see Ch. V), and Cédric Villani, at the close of the *Semaine des mathématiques* in 2015 on the theme *Les mathématiques nous transportent* (Mathematics transport us)[4], and then Paris, Lyon, Rennes and Toulouse in 2017. Within the framework of the organization of this second Forum, the CFEM took the initiative to produce a "Panorama", a collection of contributions on the theme "Mathematics and languages", theme of the *Semaine des mathématiques*. It solicited contributions from a variety of sources, with the aim of obtaining diverse perspectives on this very broad topic[5].

We will see that the CFEM also participated in the 3rd great Forum of Living Mathematics, but without piloting it as it had done for the two previous ones.

FIGURE 2. National Guidebook prepared by the Ministry

5.2. Participation in the Year of Mathematics. In partnership with the CNRS (*Centre National de la Recherche Scientifique*, National Center for Scientific Research) for its 80 years, the Ministry of National Education and Youth launched the *Année des mathématiques* (Year of Mathematics) on October 2, 2019[6]. Its ambition was to show the general public the living face of mathematics and to strengthen the link between the world of research and secondary school teachers[7]. The CFEM was invited to participate to the steering committee for this Year of Mathematics, which was to be punctuated by flagship events and meetings throughout France, involving numerous associative partners, and training activities for teachers, in line with the measures of the Villani-Torossian plan, implemented by the *Mission Mathématiques*[8]. The last event closing the Year was to be the participation of France in the ICME-14 congress. The

[4] `https://mathematiquesvivantes.weebly.com/`

[5] `http://www.cfem.asso.fr/actualites/forum-mathematiques-vivantes-2017`

[6] `https://www.education.gouv.fr/annee-des-mathematiques-2019-2020-5444` and on the CNRS website `http://annee.math.cnrs.fr/`

[7] `http://www.cnrs.fr/fr/cnrsinfo/annee-des-mathematiques-rapprocher-chercheurs-et-enseignants`, [**18**]

[8] `https://eduscol.education.fr/1476/actualites-de-la-mission-mathematiques`

Covid-19 pandemic disrupted the organization, and the *Grand Forum des Mathématiques Vivantes* (GFMV), scheduled to take place in Lyon from May 13 to 16, 2020, was postponed and transformed into a more modest and remote event[9]. CFEM has actively participated in the realization of the National Guidebook of the Year [**75**] (figure 2) and in the committee and scientific organization of the GFMV.

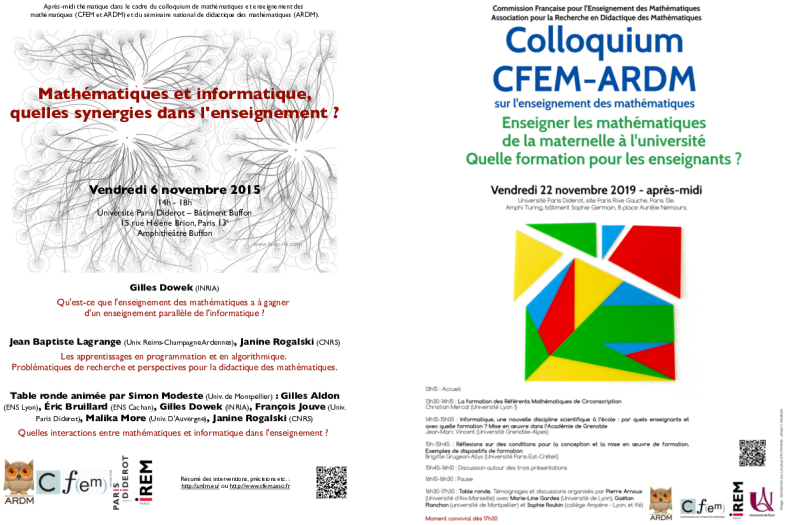

FIGURE 3. Posters of the CFEM-ARDM Colloquia (years 2015 and 2019)

5.3. Colloquium CFEM-ARDM. The colloquium on the didactics of mathematics, organized since 2005 by the ARDM, in connection with its national seminar, and the CFEM has evolved since 2012. Renamed Colloquium on Mathematics Education, it was expanded with the participation of several speakers and a round table. This event, in which didactics is still very much present, is intended to bring together the entire mathematics teaching community on a unifying theme: 'Mathematics and citizenship' (2017), 'Concrete and abstract in the learning of mathematics, from kindergarten to university' (2018), 'Teaching mathematics from kindergarten to university. What training for teachers?' (2019), 'Appropriation of mathematics and its uses in society (School, teaching and training)' (2020), and 'Recruiting mathematics teachers: what mathematical, didactic, and pedagogical knowledge and skills for primary and secondary school teachers?' (April 2021). These last two editions were held remotely, with a large number of participants.

[9] `https://eduscol.education.fr/2692/grand-forum-des-mathematiques-vivantes` and the program `https://gfmv.enseigne.ac-lyon.fr/distanciel/programmes_mercredi.html`

Let us also mention the organization in March 2018 of a day on "The teaching of mathematics, computer science and physics in the transition from high school to university: continuity or rupture?" This subject of the transition high school-university (in French transition L-U for *lycée - université*) has only recently begun to preoccupy university training programs, which were content until then to regret the "gaps" in students' knowledge in the Bachelor's program, without really taking an interest in the knowledge and skills acquired at the end of high school. The upheaval of the reform of the baccalaureate now makes it essential to think about this transition as soon as the training models are designed and to establish criteria for accepting students when they enter higher education. For mathematics, the subject has even become a major preoccupation, but it is still too early to judge the consequences of the reform, since the first generation to have undergone the reform will take the baccalaureate this year, 2021.

5.4. Communication. To maintain the links within the community between the various events organized during the year, the CFEM produces a Bulletin which was monthly under the presidency of Luc Trouche, and became biannual[10], supplemented by a monthly newsletter.

The CFEM also participated in the realization of a brochure intended for students and their families, *Zoom métiers des mathématiques de la statistique et de l'informatique* (Focus on careers in mathematics, statistics and computer science). This program is

FIGURE 4. Booklets Focus on Careers, 2015 and 2021 editions

organized in partnership with Onisep (National Office of Educational and Vocational Information), learned societies (CFEM's constituent associations) and AMIES, Agency for Mathematics in Interaction with Industry and Society. It is important to inform students of the many careers available to them after higher education (Bac - 3 to Bac +

[10] http://www.cfem.asso.fr/liaison-cfem

8). Two brochures had already been produced in previous years, and CFEM wanted to be associated with this new edition. Indeed, the question of student orientation arises in general, but particularly in high school for students who would have no difficulty in pursuing studies in mathematics, but are not encouraged to do so, because the teachers themselves, the general public and the media are not aware of the possibilities available. Particular attention should be paid to helping girls to find their way in the world of mathematics, to which the association Women and Mathematics, a member of the CFEM, contributes a great deal of energy[11].

6. What's next?

The rest of this document sheds light on a few particular points which have been chosen because they seem to us to reflect the current situation of mathematics teaching in France, seen from the point of view of the community which is dedicated to it through teaching or research. The five chapters that follow are each dedicated to an aspect: curricular evolutions (II), training and recruitment of teachers (III), research in didactics (IV), the IREM network (V) and popularization actions (in schools, extracurricular activities or for the general public) (VI). They are autonomous, with of course some intersections but nevertheless the concern to avoid repetition. This is a partial panorama and we have hardly touched on the consequences of the Covid-19 pandemic, which has already begun to be investigated and worked on (e.g., [**6**]) We will nevertheless underline the way in which the community has tried to adapt to the pandemic situation with some very fine successes and also great solidarity.

This text, written by several hands, has a critical tone at times. It reflects a concern that goes beyond the sole subject of mathematics education, but nevertheless finds its source in specific points directly related to it. Except in Chapter VI, which deals with more rewarding aspects, which are entirely managed and assumed by our community, with a great deal of freedom that allows initiatives but also networks that allow them to be encouraged and valued. Even on this subject, one can think that this activity is all the more intense as traditional teaching shows its weaknesses and despite its remarkable character and intensity, it affects only a minority of students.

The situation was complex, with a mixture of fairly clear criticism of the way in which the reforms were decided and implemented, as we shall see in Chapter III with the training of teachers, and an awareness of the need to change the structures, the contents, the professions and the training; a certain lucidity about the complexity of the task and the impossibility of satisfying everyone; but also a contestation of the place given to the sciences, and in particular to mathematics, in the core curriculum of the general high school; a certain anger at the increase in inequalities as they appear in the choice of specialities in the *lycée*, even though this situation was predicted [**112**][**113**]; a great regret that the real energy that the institution seems to want to bring to certain actions is quickly dissipated; there follows a certain bitterness from one part of the teaching staff and an incredible motivation of another part, which is nevertheless not sufficiently encouraged.

[11] `https://femmes-et-maths.fr/wp-content/uploads/2020/02/Propositions2020.pdf`

Many reports have already been written on some of the subjects we have chosen, some of these reports by the institution (reports by the IGÉSR [**84**], CNESCO [**29**], DEPP [**109**], [**110**] or Cour des comptes [**39**] or the Académie des sciences [**2**] to mention only a few). There can be no question of competing with these very detailed studies, but we encourage interested readers to go through them to gain a better understanding of the evolution of the situation while verifying a certain stability in the findings. We have chosen not to deal with the results of the international tests PISA [**123**], TIMSS [**143**], or the national CEDRE (cycle of disciplinary assessments carried out on a sample basis [**111**]) which provide the rare occasions when the subject receives media attention, and are already the subject of numerous analyses. Finally, on the reform of mathematics teaching, a National Conference on the teaching of mathematics in primary and lower secondary schools was organized by Ifé (French Institute for Education) at the request of the DGESCO in 2012[12]; several commissions did in-depth work, including on the contents for the CREM [**88**] chaired by Jean-Pierre Kahane, at the turn of the century, and the last one to date, called the Villani-Torossians commission, will be mentioned several times in the rest of the text, in particular in Chapter II. These commissions had real resources to take stock and analyze the situation, and we can rely on their work. Nevertheless, they were relatively successful, because each time, the measures they recommended were not implemented with the support of a long term program that met the needs. And the Villani-Torossian commission [**88**] does not seem to escape this observation either, despite the implementation of an ambitious associated plan.

The training of future teachers and in-service training must benefit from the contributions of research in didactics, which has reached a real maturity in France, as shown in Chapter IV. Synergies must be strengthened between fields that are still too compartmentalized, despite the role that the IREMs have played for decades in bringing them together (see for example [**30**]). A very positive point for reinforcing synergies comes from the CNRS's awareness that the research pool is fed upstream by well-trained students, which is accompanied by a greater interest in teaching.

So what do we hope for? The creation of the IREMs has been a (rare) ideal example where politics has responded concretely to projects proposed by enlightened teachers, perhaps by chance of people and social context. We are trying to keep alive the small flame of this optimism, and Chapter V reflects it. We hope that the Ministry, and its various bodies, will understand the interest of taking advantage of this existing network, which has been organized for 50 years, and will help to give it a new lease of life, encourage closer links at the level of the academies in order to co-construct training courses, and encourage teachers in the personal development that the IREMs allow. It seems to us essential that collaboration between the community linked to mathematics teaching and the institution should be encouraged at all levels, and the CFEM can be one of the actors in this rapprochement.

[12] `http://educmath.ens-lyon.fr/Educmath/manifestations/dossier-manifestations/conference-nationale`

II Recent curricular evolutions

France has undergone major curricular reforms during this century, and the reform process seems to have accelerated in recent years [**5**], [**69**]. Reforms have concerned teaching structures, syllabuses, evaluation methods and teacher training, and to these must be added the measures taken following the publication of the report prepared by the Villani-Torossian Commission in February 2018 [**149**].

In this section, after recalling the structure of the French educational system, we briefly describe the main reforms that have taken place since 2000 and underline some major trends of these, such as the increasing place given to probability and statistics or the increased attention paid to modeling and interdisciplinarity. We then focus on one of these trends: the rise of algorithmics and programming in mathematics programs, going along with the emergence of computer science as an independent school discipline. As shown by the ICMI 24 Study [**138**], these trends are shared at the international level. We then consider the Villani-Torossian Commission and its consequences, and conclude, outlining positive changes, but also expressing the concerns raised by the accumulation of reforms, and emphasizing the need for training and support for teachers, and also the need to set up effective regulation processes.

1. The structure of the French educational system

Figure 1 shows the structure of the French education system. Since September 2019, compulsory schooling begins at age 3, with 3 years of kindergarten, but 97%[1] of the children were already enrolled at age 3, 99.6% at age 4 and 100% at age 5 in 2018-2019. Kindergarten is followed by 5 years of elementary school, the combination of kindergarten and elementary school constituting primary education (about 6.7 million students). Since 2008, education has been organized in three-year cycles and cycle 3 includes the first year of secondary education since 2016-2017.

Secondary education (about 6 million students) begins with four years of middle school (*sixième* to *troisième*), in a structure, unified since 1975, the *College*. Since 2016-2017, the last three years constitute cycle 4. The structure diversifies at the high school level, the *Lycée*, with a first separation in the first year of high school (*seconde*) between general and technological high school (71.6% of students), and vocational high school (28.4%). Both types of high schools lead to the baccalaureate, a national exam, since the creation of the vocational baccalaureate in 1985. However, vocational high school also prepares students for intermediate diplomas, notably the CAP (Certificate of professional aptitude). Moreover, in 2010, the curriculum of vocational high school, originally 4 years long, has been aligned with the 3-year curriculum of the general

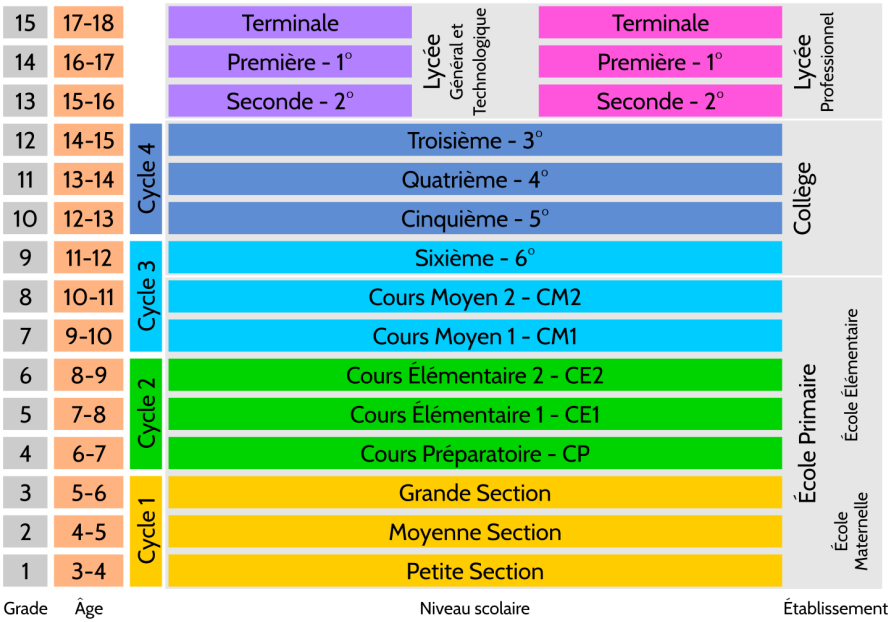

FIGURE 1. The structure of primary and secondary education in France

and technological high school. A second bifurcation takes place in the second year (*première*) between the general and technological high schools.

Until the 2019-2020 school year, in *première*, the general high school offered three series: Literature (L), Economic and Social Sciences (ES) and Sciences (S)[2]. In addition, a choice of options and specialties in the final year of high school (*terminale*) complemented this differentiation by series. The technological high school offered 8 distinct series and the vocational high school an even greater diversity. The high school reform, which came into effect in September 2019, has profoundly changed the structure of the general high school. Series have disappeared. In *première*, students follow a common core curriculum of 16h per week where mathematics is only present in a 2h science course shared between four disciplines, and they choose 3 specialties (3x4h a week) among 12 a priori possible[3]. They have to give up one specialty in *terminale* and they continue the other two with a weekly schedule of 6 hours. In the technological

[1]The numbers and percentages are taken from the publication Repères et références statistiques 2020 of the DEPP (Direction of Evaluation, Prospective and Performance) of the Ministry of National Education, Youth and Sports [**130**]

[2]A detailed description of he French educational system at the beginning of this century is accessible in (Dorier, 2004) prepared for ICME-10 by the CFEM: `http://www.cfem.asso.fr/ressources/panorama`

[3]In 2019-2020, 68,7% of students chose the mathematics specialty. It was the first specialty chosen, the second being physics-chemistry with 46,7% of students.

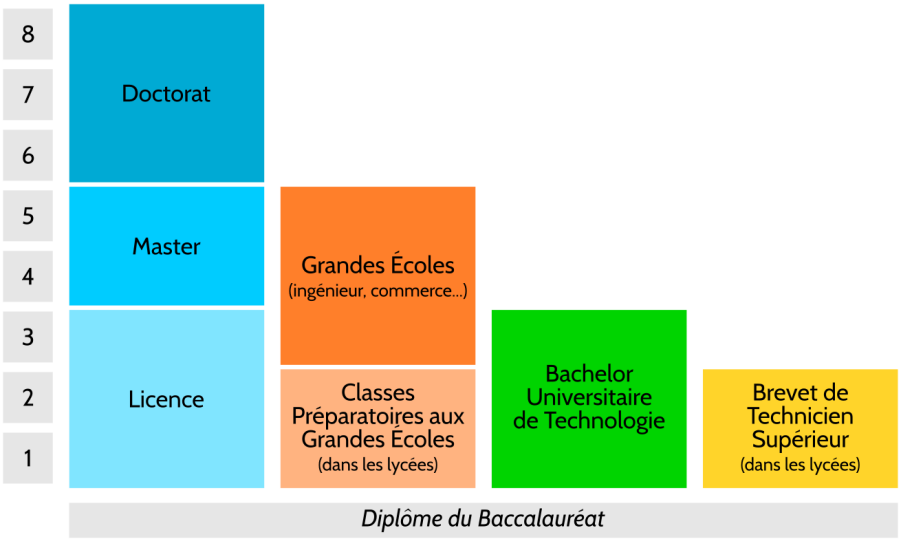

FIGURE 2. The structure of tertiary education France

high school, the series are maintained with adapted specialties. A restructuring of the vocational high school is also underway (see section 2 below).

The baccalaureate[4] gives access to tertiary education (about 2.9 million students). Non-selective and selective programs, both in and out of university, coexist at this level, including the traditional CPGE (Preparatory Classes to Grandes Écoles) and the classes preparing for the higher technician diploma (BTS) held in secondary schools, as shown in Figure 2. As in other European countries, university studies are organized into a Bachelor's degree (Licence, 3 years), a Master's degree (2 years) and a Doctorate since 2002. In addition to this already complex structure of the education system, exist institutions such as the agricultural high schools which depend on the Ministry of Agriculture and specific structures to accommodate students with different disabilities, even if these are increasingly enrolled in regular classes, as part of an education that is intended to be more inclusive (law no. 2005-102 of February 11, 2005, Peillon law of July 8, 2013)[5].

[4]The percentages of success in the baccalaureate in 2019 were 91.1% for the general baccalaureate, 88% for the technological baccalaureate and 82.4% for the vocational baccalaureate. In 2019, 80% of a generation helds the baccalaureate (42.6% with a general baccalaureate, 16.5% with a technological baccalaureate and 20.9% with a vocational baccalaureate).

[5]`https://www.legifrance.gouv.fr/loda/id/JORFTEXT000000809647/` and `https://www.education.gouv.fr/loi-ndeg2013-595-du-8-juillet-2013-d-orientation-et-de-programmation-pour-la-refondation-de-l-ecole-5618`

Traditionally, in primary school mathematics is taught by generalist teachers, in vocational high school by teachers teaching both mathematics and physics-chemistry, and in middle school and general and technological high school by teachers who only teach mathematics. These can teach at both levels.

Since the creation of IUFMs (University Institutes for Teacher Education) in 1990, which became ESPEs (Higher Schools for Professorship and Education) in 2013 and INSPEs (Higher National Institutes for Professorship and Education) in 2019, initial teacher training has been shared between these institutes and the universities to which they are attached or with which they have conventions. Traditionally, teachers are recruited through a competitive examination and have the status of civil servants. Since 2010, they must also obtain a master degree. The place of the competitive examination, of internships in schools and their conditions, the status of pre-service teachers during internships, have varied during the course of reforms and a new reform, currently underway, will again modify the system (see Chapter III). As far as mathematics is concerned, the current difficulty in attracting students to the teaching profession has greatly diversified the career paths and university disciplines of origin of candidates in recent years, and it has considerably increased the number of teachers recruited temporarily on a contractual basis, without any specific training, particularly in socially disadvantaged areas.

2. Curricular reforms (2000-2020)

Curricular reforms are regular in France. Nevertheless, their number seems to have accelerated in recent years and they concern both programs and structures. It should be noted that in France, programs are national and reforms are generally implemented at the same time throughout the country. Moreover, political changes have led, in 20 years, to the passing of three successive laws on education and the school organization: the law of orientation and program for the future of school (Fillon law 2005), the law of orientation and programming for the refoundation of the School of the Republic (Peillon law 2015), the law for a school of confidence (Blanquer law 2019)[6]. All these laws have had curricular impacts.

2.1. Primary and secondary education. The table in figure 3 cites the main reforms since 2000.

The century began with a reform of general high school the preparation of which was under the aegis of the CNP (National council for programs) created in 1989. Among the important changes brought about by this reform, we find the new importance given to the field of statistics, with an introduction to inferential statistics in *seconde*, as well as the attention paid to the relationships between scientific disciplines and more broadly

[6] https://www.education.gouv.fr/bo/2005/18/MENX0400282L.htm, https://www.education.gouv.fr/loi-ndeg2013-595-du-8-juillet-2013-d-orientation-et-de-programmation-pour-la-refondation-de-l-ecole-5618 and https://www.education.gouv.fr/la-loi-pour-une-ecole-de-la-confiance-5474

2000	Reform of general high school piloted by the CNP (National council for programs)	2012	Creation of the CSP (Higher Council for Programs)
2002	Reform of elementary school	2015	Peillon law – New common core of knowledge, competences and culture
2005-2006	Fillon law and dissolution of the CNP – Common core of knowledge and competencies – Reform of middle school	2015	Reform of kindergarten
2008	Reform of primary school	2016	Reform of elementary school and middle school
2009-2010	Reform of vocational high school (2009) - Reform of general and technological high school (2010)	2018-2020	Blanquer law – Reform of high school and baccalaureate – Reform of access to tertiary education

FIGURE 3. Curricular evolutions in France (primary and secondary education)

to interdisciplinarity. This is illustrated by the introduction of the so-called TPE (Supervised personal work), with a weekly schedule of 2 hours, in *première* and then in *terminale*. These projects, for which a set of very broad national themes are proposed, are carried out by small groups of students over a semester. They involve at least two disciplines and are co-supervised by their teachers. In this reform, we also note a differentiation of mathematics syllabuses between series, better thought-out. This has led, for example, to the introduction of graph theory in the ES syllabus.

This reform was followed in 2002 by a reform of elementary school that emphasized problem solving, problems to motivate the introduction of new concepts, to exploit them, but also to develop research aptitudes. For these two reforms, many accompanying documents were produced by the groups of experts in charge of writing the syllabuses, having the status of official resources for teachers.

In 2005, following the Fillon law, the CNP was dissolved, but a major change occurred. The idea, in gestation for decades, of formalizing a common core of knowledge and competencies whose mastery should be ensured to all students by compulsory school, entered the law[7]. This common core, published in 2006, was organized around seven pillars, one of which being entitled "The main elements of mathematics". Overall, this pillar included the main elements of the new middle school syllabus implemented the same year, but with reduced expectations in terms of expertise [**10**].

In 2008, a new reform of primary education followed, rethinking its organization with, in particular, the introduction of a 2-hour slot per week for personalized support activities, and the organization in cycles. It also proposed more concise syllabuses,

[7]`http://cache.media.education.gouv.fr/file/51/3/3513.pdf`

putting the emphasis back on calculation in mathematics and, in particular, on the learning of techniques for arithmetic operations. These syllabuses were expected to ensure that all students would master the fundamentals.

After an aborted attempt in 2008, a new reform of high school also took place from 2010. In general high school, it confirmed the importance given to the area of probability and statistics, and the attention paid to interdisciplinarity with a new optional course MPS (Methods and scientific practices) proposed in *seconde*. However, beyond the re-introduction of logic, the main novelty was the inclusion of algorithmics in the mathematics syllabuses from the *seconde* level onwards and the creation of the ISN specialty (Informatique et sciences du numérique - Informatics and digital science) in addition to the four existing scientific specialties (Mathematics, Physical Sciences, SVT (Life and Earth Sciences), Engineering sciences) in *terminale*.

The political change in 2012 led to a new law, the Peillon law, and to the creation of the CSP (Higher Council for Programs) which was asked to rethink the common core. The new common core of knowledge, competencies and culture, adopted in 2015, is organized around five transversal areas to which the various disciplines must contribute[8]. In connection with this common core, new syllabuses have been prepared, under the aegis of the CSP for primary school and middle school [69]. Their implementation was accelerated, in only two years, in 2015-2016 for kindergarten and in 2016-2017 for elementary school and middle school. In primary school, the priorities of the previous reform were reconsidered as they had not produced the expected results. In middle school, the main novelty is the introduction of a new area "Algorithmics and programming", an area whose teaching is shared between mathematics and technology.

The political change in 2017 has led to more radical changes: reform of access to tertiary education, reform of the baccalaureate, high school reform with, in particular, the abolition of the general high school series as mentioned above, and a complete reorganization of the teaching offer in mathematics. In *première* and *terminale*, in the common core students have a unique science course of only 2 hours per week. This course must mobilize physical and chemical sciences, life and earth sciences, computer science and mathematics, on cross-cutting themes[9]. In *terminale*, students who have taken the math specialty in *première* (4h per week) can keep it (6h) and even add a course entitled “Expert mathematics” (3h) (if their high school offers it); if they abandon the math specialty, they may take a course entitled “Complementary mathematics”

[8]These five areas are: 1. Languages for thinking and communicating, 2. Methods and tools for learning, 3. The education of the person and the citizen, 4. Natural systems and technical systems, 5. Representations of the world and human activity. See `http://cache.media.education.gouv.fr/file/17/45/6/Socle_commun_de_connaissances,_de_competences_et_de_culture_415456.pdf`

[9]The titles of the themes for this course are as follows: in *première*: 1. A long history of matter, 2. The sun, our source of energy, 3. The earth, a singular star, 4. Sound and music as carriers of information (`https://cache.media.education.gouv.fr/file/SP1-MEN-22-1-2019/13/4/spe573_annexe_1063134.pdf`). In *terminale*: 1. Science, climate and society, 2. The future of energy, 3. A history of life (`https://cache.media.education.gouv.fr/file/SPE8_MENJ_25_7_2019/84/7/spe241_annexe_1158847.pdf`).

(3h)[10]. Regarding the syllabuses themselves, among the novelties we note, from the *seconde* level, an official entry of the history of mathematics with a dedicated section in each of the five domains of the syllabus[11] and also, for each domain, a certain number of proofs that the students have to meet.

During this period, a significant evolution of secondary vocational education has also taken place, the last reform of which dated back to 1995. The 2009 reform aligned the duration of the vocational baccalaureate with that of other baccalaureates (3 years). The first year (*seconde*) is a common core year, then the syllabuses are differentiated according to three groups of professions, which greatly reduces the existing diversity. In terms of content, the new syllabuses are generally less ambitious than those of 1995, but the teaching of statistics has been reinforced and a complementary mathematics program is proposed to facilitate the pursuit of studies after the baccalaureate, during the hours dedicated to the personalized support of students. From the pedagogical point of view, inquiry-based practices are recommended, combined with an experimental approach to mathematics based on the use of digital technologies. Assessment of this experimental capacity counts for 30% of the mathematics mark in the exam. A new pedagogical device is also set up to ensure the link between the mathematics and specialty courses: EGLS (General teaching linked to the specialty). Without a defined syllabus and not assessed, the EGLS has constituted a real space of freedom. In the ongoing and less important current reform, one notes the introduction of algorithmics and programming in *seconde*, and the abolition of EGLS in favor of much more structured co-interventions.

2.2. The preparatory classes for Grandes Écoles (CPGE). In this section, we focus on the changes in the mathematics syllabuses of the numerically predominant CPGE scientific streams: MP (mathematics and physics), PC (physics and chemistry) and PSI (physics and engineering sciences). These syllabuses have large parts in common, the MP stream being characterized by some further developments.

Following the high school reform of 2010, the syllabuses of these CPGEs were considerably modified in 2013. Among the major changes, one notes the introduction of a probability course, centered on discrete random variables, the disappearance of a large part of geometry (conics, quadrics, parametric curves), of a part of analysis (nonlinear differential equations, diffeomorphisms) and that of Fourier series. These changes were prompted, on the one hand by the need to include a substantial introduction to randomness in the scientific CPGEs, the absence of which seemed anachronistic, and on the other hand, by the strong demand for a reduction of the syllabus coming from part of the teachers.

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[10]In terminale, in 2020-2021, 41.2% of students kept the math specialty and 17.5% chose the Complementary Maths option. Moreover, 14% of those who chose the math specialty also chose the Expert Maths option (`https://www.education.gouv.fr/39-des-eleves-de-terminale-generale-suivent-un-enseignement-optionnel-en-plus-de-leurs-deux-323285`).

[11]These five areas are: numbers and computation, geometry, functions, statistics and probability, algorithmics and programming, set vocabulary and logic – `https://cache.media.education.gouv.fr/file/SP1-MEN-22-1-2019/95/7/spe631_annexe_1062957.pdf`

Eight years later, we can evaluate this reform. The introduction of probability, which was initially greeted with mixed reactions, is now widely recognized as a great success, from the point of view of both teachers and students. The corpus of probability, although quite limited, allows for very rich interactions with the rest of the syllabus, resulting in the creation of exercises and competition problems that are simultaneously ambitious, realistic and of high quality; these address very diverse topics (random walks, Markov chains, use of probabilities in combinatorics, arithmetic, etc.). The simplifications in geometry and non-linear analysis are generally considered relevant. On the contrary, the disappearance of Fourier series is regretted from a double point of view. On the one hand, they constituted, by the diversity of the ideas involved, a very beautiful pedagogical object. On the other hand, their teaching could be modulated in a satisfactory way according to the level of the classes. Moreover, their use in physics played an important role in the dialogue between scientific disciplines.

As previously mentioned, the new high school reform modifies mathematics teaching very substantially. For students strongly attracted to mathematics, the new programs, more ambitious in their expectations as well as in their content, should a priori produce better results. However, there are still uncertainties about the long-term effects of the reform, due to its very structure and scale as well as to the disruptions created by the current pandemic. The expert panels worked with the perspective of "tyding up " the 2013 syllabuses. Thus, the change is not as extensive as that of 2013 and the content remains stable overall. Experts have worked along the following lines:

- a rewriting of the probability syllabus, to make its form more fluid and its expectations clearer, without modifying its content;
- the pruning of certain branches, too much isolated, and the addition of themes consistent with the current syllabuses to increase their coherence;
- the explicit mention, at the request of the major engineering schools, of the "Optimization" theme.

However, there is no reintroduction of Fourier series. The first year syllabuses have been published, those for the second year are in the process of being validated.

In addition, the reform establishes a new scientific CPGE stream, MPI (mathematics, physics and computer science), called MP2I (mathematics, physics, engineering and computer science) in the first year. The aim here is to provide an outlet for high school students who have chosen the NSI specialty in high school and, more broadly, to create a program in which the teaching of computer science will be strongly reinforced with respect to the existing offer, while the teaching of physics will be reduced. The mathematics syllabuses will be those of the MP stream.

In the following section, we focus on one clear trend in these multiple and diverse curricular reforms: the rise of the field of algorithmics and programming, going hand in hand with an increasing focus on the development of students' digital literacy and the empowerment of computer science as a teaching discipline.

3. Algorithmics, programming and computer science

Like many countries, France experimented with an introduction of computer science in secondary education in the 1980s, with an initiation to programming in high school. Then the software development led to the abandonment of this teaching in favor of the use of digital tools, the so-called TICE (Information and Communication Technologies for Education) in all disciplines, software (spreadsheet, word processing software...), then tools related to the development of the Internet. In mathematics, this resulted in the integration of specific tools (softwares for dynamic geometry, for formal calculation, graphic plotters, graphic and symbolic calculators...). The teaching of computer science as such has only reappeared in secondary education in the last decade [**69**].

To understand what has happened regarding algorithmics in mathematics education, one must go back to the creation of the CREM (Commission of Reflection on Mathematics Teaching) in 1999. This commission, chaired by Jean-Pierre Kahane, produced several significant reports during the following years [**88**], including a report entitled "Computer Science and Mathematics Education" recommending the introduction of some computer science teaching in mathematics courses. As early as the high school reform of 2000, we find some algorithmics, but only in the syllabuses of the L series. The reform implemented between 2009 and 2012 generalized this introduction by proposing a transversal teaching of algorithmics (and also of logic) incorporated in the different chapters of mathematics. It should be noted that no programming language was imposed, and that many software and languages have cohabited. At that time, there was a strong demand from society, scientists and industry to teach computer science to all students, and the teaching of computer science as a specialty of the scientific *terminale* (the ISN specialty mentioned above) was set up. However, this was only a first step, and the aspiration of the computer science community to make computer science a real school discipline materialized in a report of the Academy of Sciences (2013) [**3**].

In 2016, on the occasion of the reform of elementary school and middle school, computer science has entered the syllabuses from elementary school onwards. In middle school, its teaching is distributed between the disciplines of mathematics and technology. In mathematics, this gave rise to a new domain, "Algorithmics and programming" in cycle 4, and the initiation to programming is based on a block programming language, such as Scratch[12]. In 2017, the entry into high school of students who had been taught computer science in middle school required an adjustment of the mathematics syllabus of the *seconde* class, explicitly introducing programming alongside algorithmics and grounding it in mathematics. The 2017 curricular documents then advocated, without naming it, the programming language Python. In 2017 too, a new exploratory course was created in *seconde*, ICN (Computer science and digital creation), which could be continued as an optional course in *première* and *terminale*. In addition, the same year, a computer science specialty was created at the CAPES of mathematics.

[12] `https://scratch.mit.edu/`

The 2019 high school reform has generalized and made explicit the use of the Python language in mathematics in the area of "Algorithmics and Programming". While previously algorithmics was present in the mathematics syllabuses of all series, the 2019 reform has restructured mathematics into a specialty course, thus limiting the audience receiving this algorithmic teaching. At the same time, a course entitled "Digital Sciences and Technology" has been created in the common core for the *seconde* level, and a new specialty NSI (Digital and Computer Sciences) created in *première* and *terminale*. This specialty is still only offered by a few high schools (8.1% of students chose it in *première* in 2019 and 3.7% in *terminale* in 2020), but it is expected to gain momentum in the coming years. 2019 also marks the creation of a body of computer science secondary teachers with the creation of a specific "NSI" CAPES[13]. The creation of the NSI specialty in high school has not led to the disappearance of algorithmic content from the mathematics syllabuses, which suggests that the teaching of algorithmic content within mathematics could have a certain sustainability, in coherence with the evolution of mathematics already pointed out in the CREM report. However, the atomization of disciplines resulting from the high school reform does not seem very favorable to interdisciplinarity, and is likely to restrict the development of connections between mathematics and NSI.

The evolution is also visible in CPGE, since the 2013 reform. This reform has created a common computer science course for the three MP, PC and PSI streams, which aims to make students master some basic concepts of algorithmics and programming, through the learning of the Python language. In addition, MP students are offered a more advanced computer science option based on the Caml language. This evolution is confirmed in the current reform with the creation of the MPI stream.

As these curricular changes show, we observe a continuous evolution towards an integration of algorithmics and programming in education in which mathematics plays a crucial role. This evolution affects today all levels of education, and it goes along with the installation, in the educational landscape, of computer science as an autonomous discipline, more clearly distinguished from mathematics than previously.

4. The Villani-Torossian Commission and its impact

The difficulties encountered by the teaching of mathematics attested to by national and international evaluations, the notoriously insufficient training in this discipline and its didactics of primary schools teachers, of whom only a small percentage (about 15%) has a scientific background, aggravated by the 2010 reform of teacher education, and the coordinated action of the mathematics community, first led, in December 2014, to the launch of a special plan concerning this discipline, the Stratégie Mathématiques (Mathematics Strategy) plan, by the Minister of Education at the time (Mrs. Najat Vallaud Belkacem)[14]. This plan, composed of 10 measures organized around three main axes (mathematics programs in tune with the times, teachers better-trained and better

[13]Leading to the disappearance of the computer science option in the CAPES of Mathematics

[14]`https://www.najat-vallaud-belkacem.com/2014/12/04/strategie-maths-le-dossier-de-presentation/`

supported for the success of their students, a new image of mathematics), however, had only limited effects.

It was abandoned with the political alternation in 2017, but the new minister decided to create a commission in charge of proposing concrete measures to improve mathematics teaching. This commission, made up of 21 members with diverse profiles, and chaired by Cédric Villani and Charles Torossian, conducted a large number of consultations over several months, crossing the points of view and suggestions of a multiplicity of actors. It collected a large number of contributions on a dedicated website, and then produced a report structured around 21 main measures. The quality of this report, published in February 2018, has been acknowledged by the community [**149**]. In this presentation, we focus on three of these measures, whose implementation began as early as the last quarter of 2018. These are measures 14, 15, and 16, which concern the in-service training and professional development of teachers:

14. Mathematics referent: Develop in-service training in mathematics for primary school teachers. In each district, encourage professional development among peers and in teams, and appoint a third pedagogical advisor, a "mathematics referent".

15. Professional development in teams: Develop in-service training for mathematics teachers at the local level, based on confidence, among peers and in teams; promote joint observation; make common time available in the schedules; identify resource persons.

16. Mathematics laboratory: To experiment, finance and evaluate within three years, starting in September 2018, the implementation of mathematics laboratories linked to higher education and designed as places for training and reflection (disciplinary, didactic and pedagogical) for teams in at least five establishments and one career campus per academy.

In the space of one year, 1228 district mathematics referents (RMC) have been appointed for primary education, chosen mainly among district educational advisors, and a 24-day training program spread over two years, combining national and academic actions, has been set up for them. A *vademecum* has also been drafted to provide a framework for this training program[15]. In its first part, this *vademecum* also specifies the RMC's missions and the desired changes in the in-service training activities under their responsibility. It is thus specified that is expected:

- local, decentralized, trust-based and long-term training;
- work structured around groups of 6-8 primary school teachers;
- work centered on mathematical content approached in depth, in view of a confrontation with classroom practice.

In 2019-2020, these RMC accompanied 3168 constellations of 6 to 8 school teachers, who worked collaboratively on a variety of themes. A dynamic seems to have been initiated. However, there is also a great heterogeneity of situations, with sometimes

[15] `https://eduscol.education.fr/1466/referents-mathematiques-de-circonscription-rmc`

the appointment of referents with inappropriate profiles or who are already taking on many responsibilities, and also an obvious disparity in the means granted. At the end of 2020, a new directive has required that these actions be entrusted preferably to existing supervisors (pedagogical advisors) who already have supervisory and administrative responsibilities.

All three measures emphasize collaborative work among teachers. The mathematics laboratories are intended to support this collaboration, while also promoting contact with higher education. Measure 16, which calls for the creation of such laboratories, is another measure whose effects are now being felt. In November 2020, the Ministry in charge of National Education (MEN) counted 240 laboratories created in high schools. Moreover, the extension of the system to middle school has begun and the MEN aims to create 150 middle school laboratories per year over the next three years. Here again, the situations are diverse and it is difficult to know in how many of these laboratories the work is not limited to the common preparation of lessons or evaluations. However, the support of the IREM network, the investment of the mathematics community with the support of the INSMI (National Institute for Mathematical Sciences and their Interactions) at the CNRS are essential supports[16]. Nevertheless, one can fear the negative effect of the disruptions caused by the high school reform and the pandemic situation, as well as the very top-down approach to the implementation of the recommendations of the Villani-Torossian commission.

5. Comments and reflections

As we can see, the French educational system has been subjected, during the last twenty years, to intense curricular reform movements which have all impacted, more or less directly, mathematics teaching. They have affected structures, curricular organization, syllabuses, assessment modes, and teacher education. They have expressed a priori valuable and recurrent ambitions: to fight against school failure, to adapt the school to the world evolution, to improve the image of vocational education, to fight against the hierarchy between series.., but they have also been submitted to political agendas and to the resulting haste. It leads to reform without seriously evaluating the effects of previous actions, or even sometimes without letting them really be implemented, without sufficiently anticipating possible effects, without organizing sufficient support over time, without real consultation with teachers [**5**]. The current high school reform, with the legitimate concerns it raises, demonstrates it. The common core of the curriculum is strongly unbalanced in favor of the humanities, and mathematics hardly lives in the two hours of scientific teaching included in it. The diversity of choices offered to students and the resulting heterogeneity of knowledge among those choosing the specialty or complementary mathematics courses, paradoxically, make interactions between disciplines more difficult, while TPE and the MPS option have disappeared. At the same time, we observe, however, some continuous lines of evolution concerning, with an obvious coherence, the different levels of schooling. This is the case for

[16]Under INSMI agency, for example, 77 training sessions were offered in 2019-2020 but, particularly due to the pandemic crisis, only 35 could take place.

algorithmics, more especially addressed in this presentation, but also for the area of statistics and probabilities, or regarding the attention paid to the interactions between mathematics and other disciplines, and consequently to modeling. We also see, in the listening and synthesizing activity of the Villani-Torossian commission, in the voluntarism at play in the implementation of some of its 21 measures, a mobilization, an energy deployed, which tries to transcend difficulties and is a source of hope. This situation may seem paradoxical, but it undoubtedly reflects, in addition to the complexity of situations, the strong commitment of the French mathematics community at large to teaching issues, and also its resilience.

Annex: Current syllabuses

Kindergarten. (J.O. 12-3-2015)
https://www.education.gouv.fr/pid25535/bulletin_officiel.html?cid_bo=86940#ecole
Memo of service n° 2019-085, 28-5-2019
https://www.education.gouv.fr/pid285/bulletin_officiel.html?cid_bo=142291

Elementary school and middle school.
Cycle 2 – version including to the special BOEN of November 11-26, 2015 the new conditions published in the BOEN n°30, July 26, 2018
https://cache.media.eduscol.education.fr/file/programmes_2018/20/0/Cycle_2_programme_consolide_1038200.pdf
Cycle 3 (idem) :
https://cache.media.eduscol.education.fr/file/programmes_2018/20/2/Cycle_3_programme_consolide_1038202.pdf
Cycle 4 (idem) :
https://cache.media.eduscol.education.fr/file/programmes_2018/20/4/Cycle_4_programme_consolide_1038204.pdf

General and technological high schools. The syllabuses of mathematics for the *seconde* level of general and technological high schools, for the *première* level of technological high school, and for the mathematics specialty of the *première* level of general high school are defined by decrees in date of 17-1-2019 published in the special issue of the BO n° 1 of 22 January 2019.
The syllabuses for the *terminale* level of technological high school, for the specialties and optional courses for the *terminale* level of general high school are defined by the decrees in date of 19-7-2019 published in the special issue of the BO n° 8 of 25 July 2019. These syllabuses came into effect in *seconde* and *première* at the start of the 2019 academic year and *terminale* at the start of the 2020 academic year.
Seconde (general and technological high school):
https://cache.media.education.gouv.fr/file/SP1-MEN-22-1-2019/95/7/spe631_annexe_1062957.pdf
Première (technological high school):

https://cache.media.education.gouv.fr/file/SP1-MEN-22-1-2019/53/0/spe630_annexe_1063530.pdf
Terminale (technological high school):
https://cache.media.education.gouv.fr/file/SPE8_MENJ_25_7_2019/91/4/spe242_annexe_1158914.pdf

Mathematics specialty (*Première*, general high school):
https://cache.media.education.gouv.fr/file/SP1-MEN-22-1-2019/16/8/spe632_annexe_1063168.pdf
Mathematics specialty (*Terminale*, general high school):
https://cache.media.education.gouv.fr/file/SPE8_MENJ_25_7_2019/90/7/spe246_annexe_1158907.pdf
Complementary mathematics option (*Terminale*, general high school):
https://cache.media.education.gouv.fr/file/SPE8_MENJ_25_7_2019/13/4/spe265_annexe_1159134.pdf
Expert mathematics option (*Terminale*, general high school):
https://cache.media.education.gouv.fr/file/SPE8_MENJ_25_7_2019/82/5/spe264_annexe_1158825.pdf

Syllabuses of the science course in the common core (general high school)
Première :
http://cache.media.education.gouv.fr/file/SP1-MEN-22-1-2019/13/4/spe573_annexe_1063134.pdf
Terminale :
https://cache.media.education.gouv.fr/file/SPE8_MENJ_25_7_2019/84/7/spe241_annexe_1158847.pdf

Vocational high school. The new syllabuses for the CAP and seconde levels are defined by decree in date of 3-04-2019 published in the special issue of the BO n° 5 of 11 avril 2019. For the première and terminale levels, they are defined by decree in date of 3-02-2020 published in the special issue of the BO n° 1 of 6 February 2020. These syllabuses come into effect for the first year of CAP and seconde levels in the academic year 2019-2020, for the second year of CAP and première levels in 2020-2021, and for the terminale level in 2021-2022.
CAP :
https://cache.media.education.gouv.fr/file/SP5-MEN-11-4-2019/61/9/spe629_annexe_1104619.pdf
Seconde :
https://cache.media.education.gouv.fr/file/SP5-MEN-11-4-2019/26/8/spe628_annexe_1105268.pdf
Première :
https://www.education.gouv.fr/sites/default/files/imported_files/document/spe003_annexe1_1239841.pdf
Terminale :

https://www.education.gouv.fr/sites/default/files/imported_files/document/spe003_annexe2_1239843.pdf

Description of the education system (Eurydice).
https://www.education.gouv.fr/eurydice-reseau-europeen-sur-les-systemes-educatifs-3182

III Teachers' training

Reading the official teachers' competency framework [**129**] is enough to convice oneself that training teachers is a complicated process, for which it is necessary to find an equilibrium point and a sinergy between the various facets of education. In France, this exercice is more complicated, because of extreme administrative instability: teachers' training is constantly being reformed. In this chapter, we will try to describe what is done in the initial training of teachers and why the organisation is so unstable. Then, we will give a brief description of what is done done during the first three years at university for future teachers and what is done after university for in-training. We will also take a look on the training of university mathematics teachers.

1. What is stable in initial teachers' training

To become a teacher, you must pass a national or academic recruitment competitive examination corresponding to your intended profession. Successful candidates serve as trainee teachers for one year, after which they may be granted tenure and become civil servants. The possibility of registering for a competitive examination or of being granted tenure is subject to a diploma requirement.

After a presentation of the institutional actors, we will describe the different professions that allow one to teach mathematics, the competitive examinations that give access to them and the necessary diplomas.

1.1. Institutional actors. At the national level, teachers' training is under the supervision of two ministries: the Ministry of National Education, Youth and Sports (MENJS), and the Ministry of Higher Education, Research and Innovation (MESRI).

France is divided into academies. In each academy, the actors of teachers' training are the *Rectorats* and the universities. The *Rectorat* (one per academy) represents both ministries in all matters related to education and training. The universities (there may be several in each academy) are in principle independent (they control their employment policies within the limits of their payroll), but are financed mainly by the MESRI, which ensures that the regulatory framework is respected and can also direct their policies via specific financing plans.

Teachers' training is necessarily at the crossroads of the universitary world, where the students belong, and the world of the *Rectorat* who is the future employer. Whether it is a school, a national institute, integrated or external to the university, the structure where this collaboration takes place is transformed according to the incessant reforms. Currently, this role is entrusted to the INSPE [1]. There is one INSPE per academy, it is a component of one university and works in close collaboration with the *Rectorat* and the other universities of the academy.

[1] Institut National Supérieur du Professorat et de l'Education, National Higher Institute for Teacher Training and education

Finding the right balance between national and local actors, deciding how to structure collaboration within the academy, allocating human and financial resources, are issues that are found in all reforms.

1.2. Profession. Up to the *baccalaureat*, there are 3 categories of teachers likely to teach mathematics, depending on the type of institution in which they work. We refer to chapter II, figure 1

- *professeurs des écoles* are elementary school teachers,
- *professeurs des lycées et collèges* are high and middle school teachers,
- *professeurs des lycées professionnels* are vocational high school teachers.

1.3. Recruitment competitive examinations. To these different professions, correspond different competitive examinations

- CRPE: recruitment examination for elementary school teachers,
- CAPES: recruitment examination for middle annd high school teachers and its variation for teaching in private education, the CAFEP,
- CAPLP: recruitment examination for vocational high school teachers.
- *Agrégation* is also a competitive examination for the recruitment of secondary school teachers, but it has a different status from the 3 previous ones and allows one to teach in mathematics bachelor's degrees or in *Classes préparatoires aux grandes écoles*[2].

The CRPE is academic (one subject per academy, candidates take the examination in one academy and if they pass, they will be assigned to that academy) and the other three examinations are national. All of these competitions include a series of written tests. If successful, they are followed by a series of oral tests. The description that we give here refers to the framework of the competitions in force from the 2022 session [**24**].

The CRPE covers the different subjects that primary school teachers will be expected to teach. Candidates must master the primary school and the first years of secondary school curricula. Mathematics and French are the most important subjects, present in the written and oral examinations. For the written tests, candidates must also choose a subject in one of the following three areas: science and technology, history-geography-moral and civic education, or art. For the oral tests, there is also a physical education and sports test, integrating scientific knowledge of child development and psychology.

The CAPES is of course focused on mathematics. In the written examination, one of the tests is purely about mathematics, while the other is more professional: it is asked to build the pattern of a teaching session, based on documents provided to the candidates. It includes a history question. At the oral examination, candidates have to present a lesson on a given subject and pass a motivation interview. Candidates should have a good knowledge of secondary school mathematics. There is a supplementary program for the first written test: it consists of mathematics that is covered during the

[2] *Classes préparatoires aux grandes écoles* appear on in chapter II figure 2

first two years at university (analysis, algebra, probability and statistics) plus a part of geometry that is studied in the third year. The use of digital tools is encouraged, and is particularly important for the lesson test.

The PLP is organised in the same way but the written subject-related test and the lesson test are in two parts: one for mathematics and one for physics and chemistry.

The *agrégation* tests are all about mathematics. The program covered is that of a first year Master's degree in fundamental mathematics. Passing this examination guarantees a good knowledge of the mathematical bases in algebra, analysis, probability and statistics, but the part of affine geometry is reduced. Although not compulsory, the *agrégation* is an asset when recruiting research teachers at university.

1.4. Diplomas. The possibility to register for these competitions, and to be confirmed as permanent teacher, is subject to a diploma requirement which has evolved over time. Since 2010, a master's degree or equivalent is required for all the competitive examinations described here. The articulation between the Master's degree and the competitive examination, in terms of content and timing, is another issue of the successive reforms.

The standard diploma for taking the CRPE, CAPES or PLP is the MEEF[3] master's degree, which is divided into four fields. We will focus on two fields where school teacher or math teachers are trained: MEEF for primary education and MEEF for secondary education.

The diploma may also be a subject-related degree. For example, students who plan to take the *agrégation* usually enrol in a master's degree in mathematics, not in education.

Finally, there are candidates who already have a job and want to change their career. In secondary education, they are mainly engineers who want to become teachers. They represent a significant proportion of the CAPES laureates. A survey carried out among trainee teachers three years in a row at the ESPE[4] of Languedoc-Roussillon from 2016 to 2019 showed that about 20% of the trainee teachers in Mathematics were engineers changing careers. In primary education, trainee teachers are more numerous and the range of professions previously practised is wider. The same survey had made it possible to identify approximately 50 % of trainee teachers in the primary education field, who are in the process of changing career.

1.5. Training. How does one become a teacher? As a student, one accumulates knowledge, often organised around an academic discipline. In mathematics, this is theoretical knowledge, centered on algebra and analysis, where the teaching of geometry has difficulty finding its place. Teacher, beyond what they have learned in this way, must master the knowledge to be taught, and think about how to teach it. They must also be able to reflect on their practice, and ask themselves questions for which they

[3] *Métiers de l'enseignement de l'éducation et de la formation*, Teaching, Education and Teacher Training Professions

[4] an ESPE is almost an INSPE see paragraph 2.1

seek answers, by themselves, from their colleagues or in the interface literature, or even the research literature.

One of the objectives of the MEEF masters is to initiate this evolution. The training must therefore provide subject-related, didactic and pedagogical elements, combine theoretical contributions with actual classroom practice, and introduce research methods.

For mathematical knowledge, it is necessary to take charge of Klein's second transition [**153**], which is the transfer of mathematical knowledge acquired at university into knowledge useful for teaching in primary or secondary schools. This transition requires a step back from academic knowledge, it takes time. It takes all the more time as a large part of the students who enter the master's degree may not be very solid on their subject, particularly in mathematics. In this sense, one can appreciate that almost all of the CAPES program is based on the second year math program at university, but affine geometry, which is very important for secondary school, is often concentrated in third year.

For future school teachers, the question of disciplinary training is asked differently because they will have to teach many different disciplines. Mathematics and French remain the most important subjects, but the same person must also teach science, art, history and geography, physical education and sports....

Teaching in vocational high schools has specificities that are often not well-known to students. They will be maths and science teachers and will have to teach both subjects. The recent reform of the vocational education requires a close collaboration with their colleagues whose subjects are linked to the vocational speciality: they have to be together in the classrooms, in front of the pupils. They must be trained for this.

In order for the training to be based on practice, it is necessary to carry out placements in classes, during which the students gradually move from the role of observer to that of a teacher with full responsibility. Their progress is accompanied by a tutor, present in the classroom, but also by university courses providing didactic and pedagogical support.

Future teachers also need to be trained in subjects as varied as knowledge of the functioning of a school, gender equality, secularism, inclusion pupils with special needs, authority, and in different fields such as psychology and sociology.

Finally, the course includes an introduction to research. Students have to write a dissertation on a teaching-related issue. The reflection must be based on both reading articles and books and on experiments in the classroom.

The two years of the Master's degree are hardly sufficient to do all this. They can only initiate the transformation of the student into a teacher. So, it is important that, after those two years, new teachers have the desire to continue the transformation. In-service training should then take over, to accompany them throughout their career. We will discuss the subject in Section 3.3.

This multi-faceted training requires teaching teams composed of trainers having various skills and jobs: university teachers, teacher-researchers in mathematics or didactics, teachers working in national education who are involved in training at university and those who welcome students into their classrooms. All these people must think together about how to build a coherent training.

It was at the creation of the ESPE that these so-called *plural* teaching teams became necessary, whereas previously the professional aspects of training was separated form the academic study of the subject. The teams had to learn to work together, to find their coherence and balance. It took more or less time, depending on the discipline. In mathematics the transformation was quite easy: we were already used to meeting each other between teachers of all levels (primary, secondary and university) and school inspectors, partly thanks to the IREMs, and also thanks to the presence of researchers in didactics and epistemology of mathematics in the mathematics departments of the universities: by teaching to the same students, we had already started to collaborate.

1.6. Agrégation. The *agrégation* is a competitive examination of a high disciplinary level, which allows one to teach in secondary schools but also in higher education, in mathematics bachelor's degrees at university or in *Classes préparatoires aux grandes écoles* (see Chapter II, Figure 2). Historically, if they teach in secondary schools, such teachers are rather assigned to the most advanced classes in high school (that is, in the *lycée*, see Chapter II, Figure 1), and they have a reduced service because they are supposed to take more responsibility in the teaching teams. But this is not a general rule.

Taking a Master MEEF after a mathematics bachelor's degree does not allow students to pass the *agrégation*. A master's degree in mathematics is required, and some universities have dedicated courses for the preparation of the *agrégation*. A solid mathematical background is an asset for teaching mathematics, but it is not enough. Training is needed to convert academic knowledge into teaching knowledge. But this aspect is not currently taken into account either in the competitive examination or in training offered within the framework of disciplinary masters or dedicated preparation courses.

2. Constant reforms

The implementation of teachers' training is complicated because it requires the convergence of the energy of many different people in an environment driven by multiple and sometimes antagonistic forces. It is necessary, for example, to articulate a competition logic, where one has to be better than the others, with a logic of learning through collective reflection; or to make a multiplicity of actors cooperate on the one hand while they may be in a situation of competition on the other hand, (the administrative structure sometimes leads us to share hours or means under difficult conditions). In the search for the best possible solution, successive governments accumulate reforms, but the difficulties shift without ever disappearing. The instability itself could well become, in the end, the main obstacle to the success of this implementation.

2.1. Historical survey. Let's start in 2010 with the so-called *mastérisation* reform.

Before this reform, teachers training was done in the IUFMs, university institutes for teachers education. Despite their name, when they were created in 1991 they were not integrated into the universities. This integration took place in 2007. In order to register for the competitive examination, you had to have a diploma equivalent to a bachelor's degree, but the preparation for the competitive examination and the year of training that followed were not linked to a diploma.

This changed with the 2010 reform: in line with the Bologna process, teachers training was now associated to a master's degree. These masters were done in the IUFMs for primary education, and in collaboration between the IUFMs and the component of universities dedicated to a disciplinary area (for instance, the Department of Mathematics) for secondary education. The competitive examination took place during the second year of the master's degree. Although in the organisation of the training there was a lot of time to study the subject and its didactic, the short duration of the internships in schools kept the training far from the classroom. The year following the competitive examination, new teachers started working full time in the classroom. They received very limited support, which was not sufficient to meet their needs. Their entry into the profession could be very difficult.

In 2013, the ESPE[5] and the MEEF masters were created out of the will to standardise training throughout the country and to strengthen collaboration between the people working in the former IUFMs, in the other components of the universities and in the rectorate. The competitive examination was placed during the first year of the Master's degree. Once they had passed the first year and the competitive examination, the students did their internship year during the second year: paid on a full-time basis, they were half-time giving courses in schools and half-time following courses at university. They had to write a dissertation on a professional question. It was a very busy year, but it allowed the dissertation and the didactics and pedagogy courses to be based on classroom practice.

In 2020, the ESPEs were transformed into INSPEs (Institut national supérieur du professorat et de l'éducation), and this little change of title shows the Ministry's desire to take more control. One of the announced objectives was to offer homogeneous training throughout the country. But the conditions under which this reform was carried out (in an emergency that limited consultation) did not allow this standardisation to be achieved. The competitive examination will once again be placed in the second year of the master's degree, which will make the year very difficult: it will be necessary to prepare for the competitive examination, do an internship and write a dissertation at the same time. It is with a certain amount of concern for our students that we approach this new system.

2.2. Linking the master's degree and the competition. The articulation of the diploma and the competition is therefore one of the parameters of the training which is

[5] *Ecole Supérieure du Professorat et de l'Education*, National Higher School for Teacher Training and Education

modified at the time of each reform. The fact that the competitive examination oscillates between first and second year obliges us to manage transitional years, sometimes with two competitive examinations for the same cohort of students, and sometimes with two examinations in the same year.

Moreover, the administrative articulation, which consists of setting the level of diploma required to take the competitive examination, must not make us forget the articulation of contents and objectives: the two years of training in the MEEF master's program end with a competitive examination, and it is inevitable that the preparation for the competitive examination should drive the training. Although the framework documents for training and for the competitive examination should be drawn up at the same time, in order to be consistent, the Ministry publishes documents which may differ greatly on certain points, a year apart. For example, in primary education, the proportion of mathematics and french in training is much greater than in the competitive examination, and teaching staff are concerned about how they will be able to support students in these conditions.

3. The continuum: from pre-professional to continuing education

The latest reform of teachers' training insists on the so called *continuum*: training begins in the bachelor's degree, with pre-professionalization, and must continue at least 3 years after tenure.

3.1. Before the master: pre-professionalization. Intended for undergraduate students, pre-professionalization can range from a simple discovery of the profession for those who follow a classic disciplinary curriculum, to an entirely dedicated training course for those who are sure of their vocation as teachers.

To help students who intend to become teachers, universities may add dedicated courses to the traditional academic ones, e.g. courses with internships (in primary or secondary schools) or for those who plan to become primary school teachers, complementary subjects, e.g. mathematics courses in literature degrees.

There are also bachelor's degree courses entirely dedicated to the training of future school teachers: so-called multidisciplinary bachelor's degrees or bachelor's degrees in educational sciences. They can start in first year or only in third year. Starting in third year is an administratively complicated choice, but it makes it possible to welcome students who have followed a real scientific curriculum for 2 years and who have matured their project to become a school teacher. They have a good chance of passing the CRPE and will join the ranks of primary school teachers with a scientific profile, which are still too thin.

Finally, the latest creation of the Ministry is the *classes préparatoires au professorat des écoles*[6] (PPPE). Intended for students who are sure they want to become school teachers, they are carried out in partnership between the universities and the rectorates. Part of the courses take place at university within the framework of a bachelor's degree, and the other part is done in *lycée* like the *classes préparatoires aux grandes écoles*,

[6]Preparatory Courses for School Teachers

following a program framed by the Ministry. The project starts at the beginning of the 2021 academic year, with a pilot course in each academy, and is intended to expand. It is not without risk. Students who choose this path will be enrolled for five years, at the end of which they will have to take a competitive examination. There is no doubt that the successful candidates will be very well trained, but what will happen to those who fail?

3.2. At the entrance to the Master's program: pre-professionalization education assistants. Launched in September 2019, the pre-professionalization education assistant (AED) scheme consists of hiring second-year undergraduate students on 3-year contracts and entrusting them with teaching tasks in a school. The tasks are evolving from accompanied practice to taking on responsibility. The remuneration is reasonable and can be combined with grants. The scheme can therefore be attractive if it does not become an obstacle to the academic success of students, which depends on the concrete conditions of implementation (place of practice, timetable, etc.). Some problems have already arisen for students following this program while they are in the second or third year of their bachelor's degree. The problems will increase when those students will start the first year of the MEEF master's degree, in September 2021. Many students will may be find themselves with a teaching load incompatible with their studies.

This scheme follows several attempts which have all failed because, although they offer the opportunity for practical experience in a school, they do not create the conditions for pre-recruitment that would allow students to succeed in their studies and to train serenely.

3.3. After the masters: continuing education. As we have said, the two years of the Master's degree are not enough to train teachers well, it is absolutely necessary to continue after the competitive examination. Training is indeed provided for during the year of internship and the first years of the profession. The rectorates are supposed to organise it, with the collaboration of universities, but the outlines and volumes are not yet defined and all the available energy is (for the moment) devoted to the implementation of the new MEEF master's programs. It is therefore too early to talk about this. The issue is particularly important for candidates who would pass the competitive examination without attending a Master MEEF, for example *agregation* laureates or candidates in professional reconversion.

Compared with similar countries, the volume of in-service training in France is very low and not always relevant. For the past two years, the Ministry has been announcing reforms in favour of a greater supply of in-service training that is more in line with the needs expressed by teachers. Mathematics already benefits from some concrete measures, put in place following the publication of the Villani-Torossian report: work in constellations in primary school and the creation of *Labomaths* in secondary school[7]. They also benefit from the existence of the IREM network which has been an essential

[7] `https://eduscol.education.fr/1469/laboratoires-de-mathematiques` ; details on the Villani-Torossian report [**149**] and the measures discussed here can be found in section 4, chapter II.

and federating actor of the continuing education in mathematics for over 50 years. We refer the reader to Chapter V.

4. University mathematics teachers' training

It might seem surprising to think that university mathematics teachers recruited after extensive studies in mathematics, most often with a doctorate in mathematics, may need training. In France, since the 2018-2019 academic year, newly recruited teachers must follow the first year a training course for a period corresponding to one sixth of their service obligation (typically 192 annual hours equivalent TD[8]) and cannot teach extra hours during this period. They can then continue this training on a voluntary basis over the following five years, for a period corresponding over to one sixth of the annual service, that is 32 hours. The training aims at deepening pedagogical skills. Depending on the circumstances, some new recruited teachers have received teachers' training during their doctoral studies, but this is not compulsory. It is therefore in some cases more an initiation than a deepening. In general, the training is not specific to the disciplinary area(s) in which the teacher-researchers will provide their teaching. As a result, they only partially respond to the challenges that university teachers are facing. This is a rather general situation in many countries as underlined by Winslow et al. [**155**, p. 60] who add that there are substantial research results to feed training courses addressing the challenges faced by university mathematics teachers.

> « The majority of UMTE [University Mathematics Teacher Education] programs seem not to focus on specific whole courses in mathematics such as calculus or linear algebra, but to address more general levels such as inquiry-based learning in mathematics. However, research in UME [University Mathematics Education] - for example on calculus - has been very substantial and could, in the future, be used for designing specific courses on "teaching calculus" for UMTE that take this knowledge into account.

Transferring the results of research in university mathematics education to university mathematics teachers is therefore an important issue today. This is one of the objectives of the French-English bilingual journal EpiDEMES (*Épijournal de Didactique et Epistémologie des Mathématiques pour l'Enseignement Supérieur*[9]) which aims at disseminating the research work carried out in France within the GDR DEMIPS (Didactics and Epistemology of Mathematics, links with Computer Science and Physics, in Higher Education) and at the international level, in particular those conducted within the INDRUM network (International Network for Didactic Research in University Mathematics) . The journal EpiDEMES aims to build a database to feed the initial and in-service training of higher education mathematics teachers. The first issues should be published by the end of 2021[10].

[8] 'Equivalent TD', i.e. tutorial hours, is the unit to count teaching service hours in French universities.

[9] `https://epidemes.episciences.org/`

[10] About this, see section 3 in chapter IV

5. Conclusion: keeping on course

Recurrent change has a negative effect on training: no sooner do teaching teams have time to analyse a training system, to understand its strengths and weaknesses and to adapt to it, than a new reform is announced. The training have to be re-organised, based on instructions that are not always clear, but always in a hurry. The teaching teams are made of people from different institutions and the administrative structures do not always facilitate collaboration between them. Neither do emergency and fatigue. The variety of teaching teams is nevertheless a precious asset for training, provided that they have time: time for reflection is essential, as is time for collaboration.

In mathematics, we can rely on the skills and know-how developed over several decades both at local and national level, notably within the inter-Irem commission for the primary education (COPIRELEM) and for the secondary education (CORFEM). This collective reflection, with all the actors involved in teachers training, is an essential issue which we can hope will help to keep us on course.

IV Research in mathematics education

1. Research in mathematics education in France: emerging themes

The presentation of the French didactic tradition in mathematics at the ICME13 conference (Artigue et al. 2019, [**9**]) recalled the history of its emergence, highlighting in particular the role played by the IREMs. These have notably led to maintain strong links with mathematics and mathematicians (see chapter V, in particular section 2.3). Artigue et al. 2019 [**9**] also showed the importance of theories in the French tradition. The further developments of mathematics education research in France are still aligned with this tradition. New themes have been developed, or have at least attracted more attention, we give some examples below:

- Assessment: Formative and summative assessment methods and their impact have been studied within national and European projects, and models for a didactical study of assessment have been proposed ([**62**], [**135**], [**37**]);
- Algorithmics: as in several other countries, in France the recent curriculum reforms introduced the study of algorithmics within the teaching of mathematics from primary to upper secondary school. This led to studies concerning this evolution, its consequences, but also more generally the links between mathematics and computer science (e.g. [**49**], [**94**]);
- Practices of mathematicians: French didacticians make the hypothesis that getting to know the processes of mathematicians (proving, modelling etc.) has essential significance for the didactics of mathematics. New works emerge in France studying research practices of mathematicians and also of researchers in other linked sciences (such as physics, life science, computer science) (e.g. [**55**], [**64**], [**156**]).
- Students with special needs: some research in France deal with mathematics education and special education with a focus on mathematical knowledge as a central content in the educational project of cognitive-disabled students (e.g. [**11**]). Students with special needs (mathematics learning disabilities, dyspraxia disabilities) and the remedial interventions in the classroom are also studied (e.g. [**121**], [**120**], [**122**]).
- Structuration of theoretical backgrounds, such as the MWS (Mathematical Working Space) of analysis ([**90**]): the MWS is a field of investigation at the university level (focusing on the notions of convergence, optimization, modeling, e.g. [**106**]) with international collaborations (Chile, Germany and Mexico for instance). A focus is made on engineering students with the development of an online exercise database [**59**].

Moreover several researchers have addressed (1) questions related with teachers' collective work, focusing on the interactions between teachers and resources; (2) questions related with University Mathematics Education. Below we develop these two selected themes.

2. Studying teachers' collective documentation work

The collective work of teachers has been the subject of research in mathematics didactics for many years, and was recently the subject of an ICMI study (ICMI Study 25, *Teachers of Mathematics Working and Learning in Collaborative Groups*[1]). In France, work has been developed on different forms of collective work: in schools, within training courses, in groups with researchers, etc. Following on from the IREMs (cf. V), the French Institute for Education [2] (Ifé) with the support of the Ministry of Education, has set up the Associated Education Places (LéA). These are schools, or groups of schools, in which several teachers (with the support of the school administration) are engaged in research with researchers (e.g. [**83**]). In what follows we do not consider this form of work involving researchers and teachers; we focus on the work of groups of teachers designing different kinds of resources.

2.1. The documentational approach to didactics: elementary concepts. The documentational approach to didactics (DAD, [**74**]) is a theoretical approach focusing on the interactions between teachers and resources, and on the consequences of these interactions in terms of professional development. The term "Resource" in DAD comes from the work of Adler [**4**]: a resource is anything likely to enhance teachers' work. A textbook, a software, a student's production or a discussion with a colleague can constitute resources for teachers. The teachers search for resources, choose them, modify them, use the modified resources in class: all this is called the teachers' documentation work. According to DAD, along their interactions with resources for a given goal (e.g. "design and implement a course about angles for grade 7"), teachers develop a document. A document comprises recombined resources, but also a scheme of use of these resources, this is represented by the equation: "document= resources + scheme". A scheme [**148**] is a stable (but adaptable) organization of the activity, for a given goal. A scheme comprises in particular professional knowledge and beliefs, called theorems-in-action (TiA): propositions considered as true by the teacher (e.g. "The students have to learn several methods to prove that two angles have the same measure"). These TiAs are developed along the activity, and used in the activity.

During their career teachers also develop a Resource System: a structured set of resources, organized according to the different goals of their activity. The analysis of these resource systems provides deep insights into the teachers' professional activity (see [**119**] for an example of analysis concerning Chinese teachers' resource systems). DAD can be used to study individual teachers' documentation work. Nevertheless it devotes a specific interest to teachers' collective work. All groups of teachers (colleagues in school, teams of trainees in a professional development program, associations etc.) share indeed resources, and have a collective documentation work. In particular, Communities of Practice (CoP, [**152**]) of teachers have a shared repertoire: in terms of DAD, this is interpreted as a shared resource system. The development of this resource system is parallel to the development of the CoP. Even within a CoP, when teachers share their

[1] `http://icmistudy25.ie.ulisboa.pt`

[2] `http://ife.ens-lyon.fr/ife`

work with colleagues, they share the resources they designed; but they cannot share their documents, since these documents encompass TiA, in particular TiAs of which the teacher is not aware ([**118**]).

What happens when teachers have a collective documentation work? Do they develop common schemes, or at least common TiAs', under given conditions? This is an important issue, for designing professional development programs drawing on teachers' collective documentation work, and more generally for understanding teachers' work and their professional development in a context where digital tools create new opportunities for teachers' collective work. For a more detailed presentation of the theory, and a reflection coming from its presentation in 15 different languages, we invite the reader to visit the website of the "DAD multilingual" project (`https://hal.archives-ouvertes.fr/DAD-MULTILINGUAL`), where these translations are available, for example the Chinese one (`https://hal.archives-ouvertes.fr/hal-02891008/document`).

2.2. Studying teachers' collective work with DAD, examples. DAD has been used from the beginning in particular for studying the consequences of the availability of digital resources on teachers' collective design. We present here two examples of such studies, contrasted in terms of contexts. The first example [**72**] concerns the design of an e-textbook by the Sésamath association. The second concerns two teachers, who decided to prepare together a lesson about probabilities for a grade 11 class.

2.2.1. *"Extraordinary" collective work: A community of teachers designing an e-textbook.* Gueudet, Pepin, Sabra et Trouche [**72**] analyzed the documentation work of a CoP designing a e-textbook for grade 10, within the French association Sésamath. This CoP was followed up by Hussein Sabra [**134**] from June 2009 to December 2013. One of the shared goals of the CoP, at the beginning of their work was: "deciding a structure of the e-textbook". They first established together a list of 38 "atoms" (e.g. "Draw a graph compatible with a table of variations" is one atom), drawing on the official curriculum. These atoms corresponded to competencies of the new official curriculum. In the discussions between the CoP members, the researchers observed a shared TiA: "we need to support teachers for organizing their courses according to [national curriculum] competencies". After this first stage, the members of the CoP discussed how these atoms should be organized in the e-textbook. The teachers of the CoP would have liked a networking where all potential paths (considered by the teachers) were possible, but they had to give up this idea (if a notion of geometry is needed in an exercise about functions, the geometry course must be placed before this exercise). The discussions led the members of the CoP to decide that the e-textbook was going to be structured in chapters. Then each chapter will have a kernel, which is a list of techniques, with a given order.

The community developed a shared document, for the goal "choosing the structure of a grade 10 e-textbook". This document encompassed several resources, in particular the atoms; and shared schemes of use of these resources. In particular, the members of

the CoP developed common TiAs like "it is not possible to leave the possibility of any path to the users"; and "the techniques provide the coherence of a chapter". In this example the members of the CoP developed common documents, concerning the design of an e-textbook. Probably their uses in class of the e-textbook differ, because of their pre-existing schemes. But for the "extraordinary" activity of designing an e-textbook, they developed new and common documents, in particular common schemes.

2.2.2. *"Ordinary" teachers' collective work.* Gueudet and Parra [**71**] analyzed the documentation work of two experienced mathematics teachers (Valeria and Gwen) working together in the same upper secondary school. Both had in particular a grade 11 section (specialized in mathematics and economics), and they decided to design together their course about tolerance intervals with the binomial law. The authors monitored the design and implementation of this class.

We summarize here briefly the results they obtained. They identified documents developed by the teachers, and noted that in spite of their strong intention of collective work, all these documents were at least partly different. The main reason for this seems to be that, as experienced teachers, they previously developed their own individual schemes, concerning the teaching of tolerance intervals. For example, for the aim "Teaching how to find a tolerance interval with a binomial law", Valeria had developed a scheme encompassing the following TiA: "the students must learn to read the table of the binomial law and to find in it the endpoints of the interval". Gwen had different TiAs, in particular: "the technique to find the interval for the binomial law is too complex, and never assessed at the baccalaureate"; "students must learn to program their calculator". Gwen proposed to work with their students on an exercise requiring writing on the calculator a program giving the binomial law tolerance interval, once entered the appropriate parameters. Valeria accepted it and actually used this exercise. But for her this was only an exercise, the program was not used further (she did not want her students to use the calculator as a "black-box"); for Gwen, the programmed calculator was then used in each exercise requiring to find a tolerance interval. The common assessment included an exercise on tolerance intervals. Gwen and Valeria chose this exercise together; the text of the exercise incorporated a journal extract, and this corresponded to a shared TiA: "the students must learn to find information in a text". But the questions were modified, to propose the two different methods for finding a tolerance interval: with the table of the binomial law, for Valeria's class; with the calculator, for Gwen's class.

This example illustrates several facts. Firstly, the existence of shared TiAs certainly fosters common work. Secondly, when TiAs are different, even a collective work (at least during a short period) leads to different documents. Valeria developed some new TiAs during this work, but her previous TiAs remained stable, she did not want her students to use the calculator for finding the tolerance interval

2.3. Methodological evolutions for the study of teachers' collective documentation work. Within the Documentational Approach to Didactics, the development of

theory has always been closely linked with the development of methods. Following teachers' documentation work, that takes place in school and out-of-school, is challenging. DAD proposes the "reflective investigation method" [**145**]. This approach follows five essential principles: a long-term follow-up; a follow-up in and out-of-class; a broad collection of all the resources used and produced; an active and reflective participation of the teacher; a confrontation between the information provided by the teacher and their actual activity. These principles can lead to different kinds of data collection, in particular in the case of a collective documentation work. As illustrated by the two cases discussed above, the communities of teachers can have very different natures and organisation, ranging from an association of teachers working online to produce an e-textbook, and a pair of teachers working in the same school and trying to prepare a lesson together. Naturally the data collection tools depend on the kind of community, and also on the question studied. For his study about the Sésamath association, Sabra [**134**] used different kinds of data:

- Discussions in the team, collected on the distant platform;
- The resources shared on the platform by the members of the community (including the successive stages of the e-textbook they designed);
- Several members of the community filled a logbook;
- The activity of one particular member of the team was followed on the platform but also in class.

This data collection allowed an analysis of the interplay between individual resources and shared resources, and the identification of professional knowledge shared within the community.

For the second example [**71**], the two teachers already participated in the research the previous year with the reflective investigation method. New data was collected about their collective work: in particular their common preparation work (in presence) was recorded. The study of teachers' collective documentation work has led to evolutions in the methods used by the studies referring to DAD (impacting also the study of individual teachers).

Wang [**150**] proposed for example the concept of "documentation-working mate": two teachers working closely together in the same school. Following the documentation work of two such teachers, their use of digital and non-digital resources, interviewing them together and separately provides a privileged access to teacher knowledge and to its evolution. Rocha [**132**] also uses this method; moreover she introduced the notion of "documentational trajectory", defined as "a path (with continuities and ruptures) linking professional events (individual and/or collectives) lived by the teacher" [**145**, p. 1245]. The analysis of teachers' documentational trajectories, during their whole career, evidences with a new perspective the importance of collective work in teachers' professional activity.

During the "Re(s)sources Conference" that took place in Lyon in 2018[3], and in the book entitled "The 'Resource' Approach to Mathematics Education" [**144**] that followed, many contributions concerned teachers' collective work and its consequences. This was an international conference, and DAD is not only a French theory; nevertheless it belongs to the French didactic research tradition.

3. University mathematics education

In the current French context, official texts have highlighted the need for training for higher education teachers (this training is not yet generalised at this stage). Work in progress should lead to professional development programs for these teachers.

In France, research at university level in Didactics and Epistemology of Mathematics and its links with Physics and Computer Science (a network called DEMIPS[4]) has been structured into a CNRS (Centre National de la Recherche Scientifique) GDR (Groupement de Recherche) involving 12 universities (see Ch. III, Section 4). DEMIPS is linked with the International Network for Didactic Research in University Mathematics (INDRUM, [**51**]). Furthermore, DEMIPS is at the origin of the journal EpiDEMES[5]: this journal aims at disseminating research results to all teachers of mathematics in higher education[6].

The objectives of DEMIPS are threefold: to develop new ways of teaching mathematics at university through collaborative work between didacticians and mathematicians; to study the interactions between mathematics, physics and computer science; to federate new research and to structure collaborations in France with research teams in education and in mathematics, physics and computer science. Three DEMIPS themes deal with mathematical contents: analysis and links with physics; linear algebra and abstract algebra and their links with physics and computer science; discrete mathematics and interactions with computer science. Two other transversal themes study reasoning and proof with a particular focus on language and logic and the practices of university teachers in mathematics and physics. Common research questions include student difficulties, the transition between secondary and university levels, the pedagogical practices of lecturers and the design of resources (didactic engineering) in university mathematics. In all these works, mathematical contents and skills are central. Moreover, one of the particularities of the French educational system is the Classes Préparatoires aux Grandes Ecoles (CPGE, subject to demanding national competitions) ([**91**] ; [**92**]): these specific institutions are now also investigated by mathematics didacticians. In addition, work on mathematics for engineering education is growing (e.g. [**124**] ; [**125**]).

In the above-mentioned research conducted in higher education, the experiments and objects of study are varied: study of teaching practices through in vivo observations (lectures and tutorials), video recordings and interviews; study of potential effects

[3] `https://resources-2018.sciencesconf.org`

[4] `https://demips.math.cnrs.fr/`

[5] `https://epidemes.episciences.org/`

[6] see also Sec. 4 in chapter III.

on student learning; didactic and cognitive study of student activity in class (lectures and tutorials) and outside class (observation of pairs or small groups of students in problem-solving and proof development situations), didactic engineering and interventions in different fields and in various contexts (beginning of higher education, teacher education, engineering education, training of new lecturers).

3.1. Study of mathematicians' practices. The study of mathematicians' practices has been the subject of research for several years, at the international level, in order to envisage new perspectives for training and teaching (e.g. [**23**] ; [**95**]; [**127**]; [**151**]).These works, demonstrating a fruitful collaboration between mathematicians and didacticians ([**64**]) allow us to claim that the study of the work of mathematicians enriches the epistemological knowledge of didacticians and opens up tangible perspectives for the design of teaching and training devices, in particular at the transition from secondary to higher education and in higher education. In this respect, the works of Gardes ([**60**]), Modeste ([**104**]), Ouvrier-Buffet ([**115**] ; [**116**]), Yvain ([**156**]) enrich the field with Epistemological Studies of Contemporary Research Practices (ECRP). Their methods are similar and allow for the development and enrichment of didactic questioning along two axes: the development of teaching situations that allow students to experience the mathematical activity studied, and the development of tools to describe and analyse student practices related to this mathematical activity.

3.2. Didactics of analysis. The field of analysis is the source of many persistent difficulties at university, already well-known in the national and international mathematics education community and often updated ([**8**]; [**126**]; [**89**]; [**20**]; [**154**]; [**73**]; [**28**]; [**61**]). These difficulties persist and the transition from secondary to higher education is particularly problematic. The specific features of the work in progress (within DEMIPS in particular) are thus to question the continuities and ruptures between secondary school and university, to take into account the links with physics, and to explore the new digital resources. Recent works are interested in this respect, for example, in the links between analysis and other mathematical domains, in particular algebra ([**147**]), geometry and probabilities ([**44**]). Also studied, in analysis, are the issues of entry into proof with its specific modes of expression (modes of reasoning, formalisation, quantification) and the problematic links with order, numerical approximation, intuition, visualisation ([**21**]; [**133**]; [**27**]).The role of semiotic representations, the understanding of relations between punctual, global, local and infinitesimal points of view on the representations of the objects of analysis, with in particular the place of digital tools and their role in teaching and learning are also the subject of recent work ([**105**]; [**19**]). We can also note work on the role of the conceptualisation of the continuum (in particular the completeness of the set $\mathbb{R}$ of real numbers with respect to the usual topology of order) in the appropriation of the concepts of analysis taught in the bachelor's degree ([**17**]; [**54**]).

3.3. Didactics of linear algebra and abstract algebra and interface with quantum mechanics. The didactics of linear algebra has been the subject of much work in France, particularly in the 1990s ([**46**]), taking into account the FUG nature of

knowledge (Formalizing, Unifying, Generalizing, [**131**]) and the importance of frame changes. At the international level, various studies have been conducted (e.g. [**47**]; [**77**]) evidencing the recurrent difficulties of students. In recent work in France, the frameworks of semiotics and Theory of Didactical Situation TSD have been jointly mobilised to study students' reasoning on the notion of linear application in the first year of CPGE ([**91**]). With regard to abstract algebra (algebraic structures), where there is little work to date, Hausberger proposes a research program using the Anthropological Theory of Didactics (ATD, [**80**]; [**81**]) and an epistemological framework dedicated to structuralism ([**79**]). Within this stream, specific results on the concept of Ideal have recently been presented ([**87**]). Algebraic structures are an interesting object of study at the crossroads with quantum mechanics ([**12**]), in an interdisciplinary perspective questioning the circulation of knowledge between disciplines and fighting against disciplinary isolation at the university, both at the undergraduate level and at the secondary-senior transition ([**93**]).

3.4. Didactics of discrete mathematics and interfaces with computer science. Still in a perspective of decompartmentalization, we can note the work at the interface between computer science and mathematics, interested in discrete mathematics, arithmetic and algorithmics. These fields of discrete mathematics are booming and are at the heart of many recent and current technological transformations in society (cryptography, automatic data processing, networks, etc.). However, discrete mathematics has a very variable place in undergraduate mathematics and computer science curricula ([**1**]; [**142**]). In France, the recent profound evolution of secondary school curricula makes the didactic question of the interaction between computer science and mathematics in higher education all the more important. Mathematicians and didacticians, at the international level, underline the importance of discrete mathematics for teaching and teacher training ([**78**] ; [**41**] ; [**45**]), in connection with the recommendations of experts societies (such as the MAA or the SMF). In France, research in mathematics education specifically investigates the teaching and learning of algorithms ([**104**]), as well as the contributions of computer science and programming to mathematics teaching ([**38**]). Discrete mathematics involve specific objects and types of reasoning that have been highlighted by fruitful collaborations between didacticians, mathematicians and computer scientists ([**63**] ; [**114**] ; [**117**]) notably within the *Maths à Modeler* Research Federation[7]. Moreover, arithmetic has already been the subject of epistemological and didactic work in secondary education and in the transition from secondary to higher education ([**15**] [**16**] ; [**60**] ; [**128**]): this work is continuing. Didactic studies have focused on the particular case of teaching graph theory in high school and university in France ([**25**]). At the centre of didactic concerns today are issues relating to the development of:

- an epistemology of mathematics and computer science in order to feed the existing work in the didactics of mathematics in higher education but also to

[7] `https://mathsamodeler.ujf-grenoble.fr/`

contribute to the development of the didactics of computer science ([**7**] ; [**56**]) for higher education,

- specific work on the learning of reasoning and proof, a fundamental issue in the transition from secondary to higher education ([**73**]) and a point of discussion at the mathematics-computer science interface ([**49**]).

3.5. Proof, logic, language and reasoning. The question of proof, logic, language and reasoning is transversal to all the mathematical themes mentioned above and proves problematic in higher education ([**136**]). Work on proof contributes significantly to the learning processes of advanced mathematical knowledge ([**53**]). Many studies, both in the French-speaking world and internationally, highlight that students cannot rely on a good mastery of the logical knowledge and skills necessary to face the formalisation and the complexification of the logical structure of mathematical statements ([**137**]; [**43**]; [**68**]; [**26**]). But the role of mathematical logic in learning to reason and prove is one of the elements on which positions diverge ([**50**]). Ongoing works study how teachers manage the many implicits (shared by the mathematics community) that are necessary for reasoning and communication ([**14**]; [**76**]; [**151**]; [**57**]; [**99**]) but are not necessarily recognized by students ([**48**]). Language studies are also evolving towards case studies in the particular context of plurilingualism, which can reinforce learning difficulties, but which can also be a resource for teaching (thematisation of cultural and language issues in courses) ([**52**]).

3.6. Study of University Teachers Practices in Mathematics and Physics. Research in didactics concerning the practices of higher education teachers is only beginning to develop, both internationally and in France, in mathematics and at the interface of other scientific disciplines ([**22**] ; [**65**]). This work has led to identify certain categories of mathematics teaching practices, in particular for lectures, which constitute an important specificity of the university ([**108**]; [**86**]). Other studies ([**98**]; [**70**]) have focused on the uses of certain teaching materials, in paper or digital form, and the consequences of these uses. A recent study conducted in a physics department revealed some features of professional identity of physics lecturers questioned about their teaching practices and the difficulties underlying them, among which the insufficient mathematical knowledge of students ([**42**]).

It is recognised that teaching mathematics to non-specialists poses specific difficulties, with mathematics being a major cause of failure for these students ([**82**]). In this context, researchers have set themselves the main objective of analyzing, characterizing the practices of higher education teachers of mathematics and physics in various institutions both in teaching situations and in the use and design of resources.

Research in didactics of mathematics in France has maintained its specific attachment to mathematics and to the mobilisation of theoretical frameworks grounding the research process, while developing new directions as we have illustrated above. It also retains its involvement in an international network, particularly with regard to the supervision of PhDs.

V The adventure of the IREM

1. General presentation of the IREM network

The IREM (IRES and IREM&S) (Institutes for Research in Mathematics, Sciences, or Mathematics and Sciences Education) network was created in France fifty years ago. Since then, the IREM have contrived to be one of the most active and unavoidable actors in mathematics education in the country, by responding to the three missions entrusted to them:

- research to improve the teaching of mathematics in non-hierarchical groups of university researchers and teachers of the first or second degree school or in higher education;
- Train teachers, especially using the results of network research;
- Disseminate the results of research in mathematics, mathematics education or history.

These missions, and the originality of their universitary structure, associating all the actors of the world of education in mathematics, are at the origin of many productions. Those works are always close to the concerns of the on-site professors, adapted to the teaching issues arising from the many curricular evolutions and to the emergence of new technological tools.

The IREM, each with their local specificity, but with constructive scientific coordination, form a network that is both adaptable and coherent. They are recognized in France and in many countries, particularly French-speaking countries, or having natural research relations with France. Some of those have in turn created or are seeking to create IREM-type structures. The IREM network is plebiscited both by researchers in mathematics and didactics and by on-site teachers. In particular, the latter use in confidence the resources developed in the network and disseminated by an evolving system of common or local media which will be detailed in this text. It is now used as a model for improving the teaching of other sciences, in particular STEM (Science, Technology, Engineering and Mathematics), which is being increasingly welcomed in the network. This is an explicit demand from both the Ministry of National Education, the scientific universities, and the researchers in didactics of other sciences.

The fiftieth anniversary of the IREM network, which was celebrated during the 2018-2019 school year, was the occasion for a broad introspection on the work carried out during these fifty years, and which still appear to us today to be quite relevant as a whole ([**58**]).

1.1. The fundamental principles of the IREM. The second half of the twentieth century has been marked, in France as elsewhere, by the need for a strong renewal of mathematics teaching, to adapt to the considerable evolutions of the knowledge and the conceptions carried out by the research in this field. The IREMs were thus born

of the conjunction of a great disarray of the teachers, following the so-called "modern mathematics" curricular reform [**40**], and the events of May 68, shaking among others the academic world. This creation, by the French government, was solicited both by the Association of Mathematical Teachers of Public Education (APMEP), which was already developing training and resources to anticipate and support the "retraining" in mathematics teachers, and by faculty-researchers aware of their responsibilities. Many mathematics teachers then brought in their enthousiasm for this militant work, close to their professional concern. The IREM then furnished them, inside the universities, with a new working environment, stimulating, within groups, away from hierarchical constraints.

The three primary missions of the IREM (see above), underpinned by the conditions of their creation, have thus led to fundamental constants, in time and space, in their functioning:

- A research activity inside universities, throughout France, within working groups of "research-action". These groups associate researchers and teacher-researchers in mathematics, mathematics education or mathematics history, as well as field teachers, each of them bringing in his own knowledge and expertise, sensitivity and questioning. This diversity of status and the reciprocal trust allow for productive classroom observation and critical thinking, and important professional development for all members. These research works are supported by universities, by local authorities (rectorates, inspectorates) and national authorities (Ministries of Education or Higher Education and Research), by the provision of work spaces, financial and, above all, human resources. (see 2.1 for details).
- Training for all teachers "from kindergarten to university", proposed by working group members, ensuring subtle consideration and approriation of work proposals. The contents are from research of the intervening group and/or other IREM groups with whom they may have interacted, or from theoretical research, suggested by the researchers of the group (see chapter IV). Training to competitive exams for internal promotions is also provided. Pre-service training of professors is impacted thanks to the presence in the groups of trainers from university institutes in charge of these training courses, who often began as trainers for their IREM.
- A multiplicity of modes of transmission of research results: direct training; drafting of research articles, publications in interface journals, publication of booklets, books, various resources, in paper and/or online; provision of books and resources in IREM or academic libraries, on IREM sites or on the network site[1].

  The network publishes four peer-reviewed journals (see 2.2 for details): *Repères IREM*, *Grand N*, *Petit x*, *Annales de Didactique et de Sciences Cognitives*.

[1] `http://www.univ-irem.fr/`

The resources on mathematics education are reviewed on the *Publimath*[2] server (co-managed with the APMEP) by creating reading sheets and linking them to available digital versions of the resources.

- The organization of conferences, either local, national (4 to 6 per year gathering 100 to 250 people each), or even international conferences on mathematics education or their history (for example the organization of HPM 2016, by the IREM of Montpellier[3], or the network international congresses, see 2.7); the organization or participation in numerous events to disseminate and promote mathematics to scholars and to the general audience: Mathematics Week (annual, national), fortnight of the Science Festival (annual), Living Mathematics Forum in 2015 and 2017 (see Ch. I, section 5.1), Mathematics Year in 2020 (section 5.2), mathematics game fairs, class hosting, internships and extracurricular actions... (see 2.6 and Chapter VI).

1.2. A strong synergy inside the network. The synergy between the different IREM, each with its own status, type of local integration, but no centralized management of the structures, is ensured by three types of assemblies, bringing together IREM members and representatives of other bodies.

- The IREM Directors Assembly (ADIREM). Four times a year, it gathers the directors, the chairman of the scientific committee, and representatives of friend associations: CFEM, APMEP, SMF, SMAI. It coordinates administrative and political aspects of the network and represents it in front of national authorities.
- The Scientific Committee of the IREM (CS-IREM). It is currently made up of 20 people, half representing the IREM and half external observers (including non-mathematicians). Its role is to observe the network's activity, evaluate its work, generate perspectives and contribute to the IREM's voice. At each of its 3 annual meetings, it organizes debates on current topics concerning mathematics education.
- The Inter-IREM Commissions (CII). They bring together members from different IREM to work on given themes, compare the work of IREM, encourage specific research, produce common resources, organize conferences and meetings, and reflect and anticipate curricular developments. They are a source of proposals thanks to their expertise for national or international actions.

  There are currently 13 CII of several types (the number and themes evolve regularly according to needs):
  - 6 of them are associated with a sector of the educational system or of the pre-service training: CII *Collège* (junior high school, grade 6 to 9), CII *Lycée* (high school, grade 10 to 12), CII *Lycée Professionnel* (vocational

[2] `http://publimath.univ-irem.fr/`

[3] `https://hpm2016.sciencesconf.org/`

high scool), CII University; secondary school teachers' training (CORFEM), primary school teachers' training (COPIRELEM). Those two last commissions organize an annual congress for teachers trainers;
- 3 of them are working on transversal themes: CII Epistemology and History of Mathematics (which organizes a congress every 2 years), CII Didactics of mathematics, and CII Information and Communication Technologies for Teaching (TICE);
- One is specialized in computer science (CIII) (currently partially taught by mathematics teachers);
- 2 are dedicated to the network resources sharing: CII Publimath, shared with APMEP, which manages the *Publimath* server, and CII *Repères-IREM*, which is the editorial commitee of the journal;
- And the new International CII (see 2.7).

The number of CII evolves according to needs. The commission for the poularization of mathematics (CII Pop'Maths) stopped in March 2019 following the publication of its book Panoramath'7 ([**33**], cf 2.6). Other commissions will soon have to be created, following the increasing opening of the IREM to groups and activities concerning other sciences (sometimes without any mathematician).

The network (the ADIREM or some CII) organizes 4 to 6 national or international symposia each year, in which network members and external persons participate. These congresses are locally organized by one of the IREM of the network. For instance, in 2019, ADIREM organized the 50th anniversary symposium in Besançon, the CII Epistemology and History organized a congress in Poitiers, registered in the national training plan for teachers (PNF)), and the annual congresses of the CORFEM and COPIRELEM, were held respectively in Strasbourg and Lausanne (Switzerland).

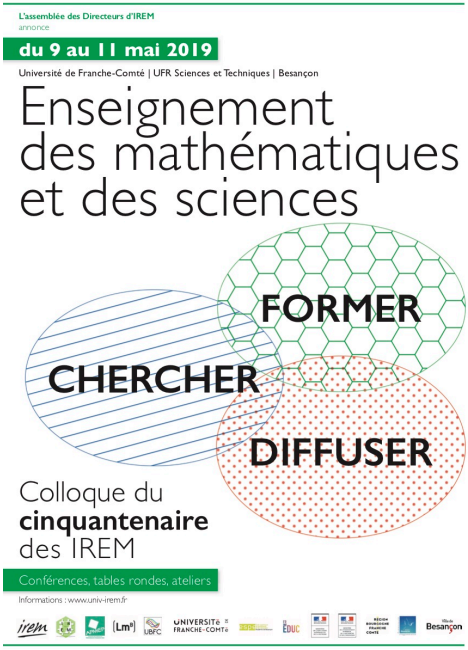
In 2020, in addition to the CORFEM and COPIRELEM colloquia, which were to be held respectively in Strasbourg and Annecy, a symposium on "Mathematics and Language Education" in Clermont-Ferrand in May and a symposium of the CII TICE in Marseille in October were cancelled. Those of CORFEM and COPIRELEM have been postponed until spring 2021.

In 2022, in addition to the CORFEM and COPIRELEM colloquia, the CII Epistemology and History colloquium will be held, and probably a colloquium of the CII *Lycée*.

Proceedings of the conferences are published and put online on the IREM network site, accompanied by some video recordings of interventions.

1.3. Insertion into the world of mathematics education. ADIREM is a member of the CFEM, and we first describe here some specific interactions between the IREM and other CFEM members.

The APMEP, which contributed to the creation of the IREM, continues to edit or co-edit some of the resources from the groups. Many exchanges take place during



IREM symposia and "APMEP Days", training congress voluntary organized by the APMEP members[4]. The IREM, thanks to their university structure and local connections, organize congresses for MATh.en.JEANS or specific actions for Women & Maths (cf 2.6 and VI). They also organized an annual challenge[5] with the French Statistical Society (SFdS).

The ARDM members are mostly also members of IREM groups that benefit from their theoretical knowledge and provide them with a strong anchorage within the field (cf 2.3 and chapter IV).

The network has been associated several times with the French Academy of Sciences for works or conferences on teacher training. In 2018, a symposium on teaching at primary school[6] was co-organized with the foundation for sciences education *La Main à la pâte*, emanation of the Academy of Sciences, which relies locally or nationally on the IREM's expertise for its activities with schoolchildren and teachers training in mathematics, and in return disseminates its resources. The network participated in the work and fed into the reports of the CREM ([**88**]), which was chaired by the academician Jean-Pierre Kahane, President of the IREM Sientific Committee from 1999

[4] https://www.apmep.fr/-Journees-Nationales-

[5] http://irem.u-bourgogne.fr/challenge-graines-de-sondeur.html

[6] http://www.fondation-lamap.org/fr/page/65880/colloque-sur-lenseignement-des-mathematiques-a-lecole-primaire

to 2003. This commission was created in 1999 by the Ministry of Education to work upstream of mathematics programs (already mentioned in chapters I and II).

The Inspectorates of Mathematics Teachers, national (IGEN, which became IGÉSR) and regional (IA-IPR), who contribute to the development of curricula and teacher training, have been very abounded by members of the IREM groups, thus participating in the irrigation of the mathematics teachers by the works produced in network. Johan Yebbou (IG), and Kadir Kebouchi (IA-IPR, Versailles), are members of the CS-IREM. Resources have been co-produced by the General Inspection and the IREM and thus have the status of official resources ([**100**], [**101**], [**102**], [**103**]). Xavier Buff, then Director of the IRES of Toulouse, participated in the work of the Superior Council of Programs (CSP), which defined the 2015-2016 programs currently in force in primary schools and junior high school (see Ch. II). The IREM have been consulted with the CFEM before the implementation in 2019 of the new high school and vocational high school programs for the final year, and the readjustments for the former year. (Ch. II).

The IREM were also interviewed by the Villani-Torossian Commission for its official report [**149**] on the teaching of mathematics in France (see ch. II, section 4). This report has repeatedly pointed to the IREM's preponderant role in teacher training. Christian Mercat, then director of the IREM of Lyon, was a member of this commission. Several proposed measures are explicitly based on the IREM, and in particular the two measures currently in place: circonscription referents in mathematics (RMC) and the mathematics laboratories in the high schools ("labomaths"), which function in a way similar to the IREM groups. Members of the ADIREM (Anne Cortella, François Recher) have participated in the drafting of the vademecums with the national mathematics mission. The IREM are particularly active in supporting the implementation of these two measures, and a number of these new missions are being entrusted to members of IREM groups. Some of the training courses of the RMC – local or national – are entrusted to the members of the groups, mathematics laboratories have been created by other members. The IREM participate with other actors in the 'Mathematics Clubs' (cf 2.6 and Ch. VI).

Finally, ADIREM is part, with the CFEM and the APMEP, of the steering committee for the Mathematics Year 2020 (extended to 2021) (see ch. I, section 5.1). This Year focused on the training of mathematics teachers, while highlighting all types of actions implemented on the territory and fostering students' interest in mathematics.

1.4. A responsive network in permanent evolution, but permanently threatened. Being universitary institutes, the IREM are flexible autonomous structures. They have kept up the militant momentum of their creation and continue to anticipate, criticize and/or support perpetual developments, in particular curricular reforms (Ch. II), and pre-service training reforms (Ch. III), affecting the teaching of mathematics in France. The Scientific Committee, through its debates, the international network, by the diversity of its experiences, the CIIs and the research groups through their work, are the strength of the IREM in the face of these developments (cf. Ch. II).

The current developments, in particular resulting from the Villani-Torossian report and relating to in-service teacher training, are largely supported locally and nationally

by the IREM, as key players capable of making the link with all the professionals affected by these changes, and having expertise on their adaptability (cf. 1.3).

The IREM also act together with the APMEP and the learned societies (SMF, SMAI, SFdS) within the CFEM in the implementation of new programs and working organization in high school: for example against the near absence of mathematics in the new common core of education, and to make it possible to choose a different mathematical content for students who are not destined for scientific studies[7].

The CII dedicated to pre-service training (CORFEM and COPIRELEM) participate in the reflection on the evolution of this training (Ch. III) and give their opinion on the reforms put in place[8].

At last, the network is very concerned about the aggravation of the unequal character of French education, particularly in mathematics, as evidenced by international evaluations. It is also concerned about the growing defection from scientific studies and the lack of students for mathematics teaching careers. In addition to its core missions, the network develops actions to disseminate and promote mathematics and scientific careers for the youngest (section 2.6), and in particular those with a low social level and towards young women with the association *Femmes et mathématiques*.

Despite all the benefits, in 50 years, the IREM have had to constantly defend their structure and their original view of mathematics education, both as a field of research and practice, and their collaborative conception of professional development and resource production. Thanks to the mobilization of many actors and the solidarity within the network, they have always managed to overcome obstacles and have survived, despite the continuous threats they face.

The means allocated to the IREM, both in terms of remuneration for their members and in terms of operations, are currently extremely limited. Working in a research group is more militant than remunerative, and the IREM only survive by gathering scant resources from all their interlocutors, always under the threat of their reduction. The "priorities" announced in recent years by the Ministries of Education for mathematics consist almost exclusively of redeploying the means allocated to this discipline to new schemes, admittedly very interesting, but without considering their cohesion with the network. Over the last three years, the resources of some IREM have been divided by three at the local level, and the start-up of other IREM has been compromised due to a lack of means.

The IREM network is today a dynamic and mature structure, constantly questioning itself but seeing in the international evolution an encouragement to pursue with this vision.

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[7] several cosigned texts in this sense, e.g. `http://www.cfem.asso.fr/actualites/communique`

[8] see for example `https://www.copirelem.fr/2021/02/27/crpe-2022-la-copirelem-alerte-le-reseau-des-inspe/` and `https://www.copirelem.fr/2021/02/11/crpe-2022-epreuve-orale-de-mathematiques-une-reflexion-sur-les-modalites-dorganisation-de-lepreuve/`

2. Detailed description of the work

The IREM were born out of the strong need for in-service training for mathematics teachers due in the 1960's by the massification of secondary education and the "modern mathematics" reform, under the decisive impulse of APMEP.

The IREM's action takes many forms: research in working groups and CII with local dissemination of their work; publications developed in an IREM or in the network; strong links with research in didactics, epistemology and history of mathematics; impact on initial teacher training; the network's involvement in curriculum development and concern; actions to popularize mathematics; international contacts; openness to other sciences. All these actions involve a large number of people, academics, professors, teacher trainers, inspectors.

2.1. Working groups and direct dissemination. No one works full time in an IREM, and IREM's achievements owe much, and more and more, to the commitment and volunteer work of IREM members. These constitute an open and moving community as new groups are regularly created while others end their projects. Participating in IREM activities stays the essential mode for mathematics teachers professional development, and also for teachers educators professionalization. Many of them, as for inspectors of the education system, have benefitted from this system.

Researches on mathematics education in the IREM are carried out in working groups, whose members (called "facilitators") are teachers (primary, secondary) and researchers in mathematics, didactics, history or epistemology of mathematics, but also teachers and researchers in other disciplines (computer science, physical science, technology, biology, economics, philosophy). These groups can be considered as places of training and professional development for all their members. The research gives rise to numerous local or national publications (see 2.2).

In 2019-2020, the 2 074 facilitators of the 27 IREM gathered regularily, with very small compensation, in 287 research groups: primary school teachers (10%), junior high school (25%) and high school teachers (about 29%), university faculties (about 27.5%), and teacher support staff (4,5%). The 13 Inter-IREM Commissions (CII, cf. 1.2) have gathered 220 of them in order to organize, distribute, exchange on the groups work and produce synthesis or original resources.

The in-service training courses offered to teachers in the IREM are run by the members of the groups, which ensures their scientific and pedagogical quality. They can rely on the many publications resulting from the work of their group or other groups, their IREM or another IREM, as well as on the synthesis work resulting from CIIs (booklets, books, conference proceedings), and on the work of didacticians or epistemologists from the network (see 2.3).

In 2019-2020, despite cancellations due to covid-19, 214 training sessions, spread over 1 555 half-days, received 7 591 trainees, for a total of 36 398 half-days of trainee training (hDTT), 165 of which were officially approuved by local or national authorities of education. 10% of the secondary school teachers, fewer elementary school teachers

were reached. 22 actions concern pre-service training. Some IREM also support preparation for body change competitive exams (12 internship in 2019 for 300 trainees and 7 500 hDTT).

In addition, training is provided at national and local congresses, seminars, or conference cycles, by an IREM, the ADIREM (Assembly of IREM Directors, see 1.2) or a CII. In 2018-2019, six national congresses for 13 days brought together 800 delegates and more than fifty conferences and seminars were also organized.

It should be noted that the demand from teachers, who highly appreciate the training provided by the IREM, as well as the supervision they offer, is for a substantial increase in internship training days. But the resources granted by the administration for in-service training are constantly decreasing, and IREM internships are often cancelled when curriculum reforms occur, and replaced by informations about the new curricula. Also, except for preparation for body change competitive exams, only short-term internships are granted (1 or 2 days, or online) when a longer time should be required to allow a better impact. The IREM network is campaigning, with the help of other societies, in particular the APMEP and the Academy of Sciences, for whom it is a priority objective, to significantly increase the possibility of in-service training for teachers. Nevertheless, online training courses (mooc, m@gistère) that are resuming the internships are offered in conjunction with the IREM to allow more time: for example, the mooc e-fan[9] at the IREM in Lyon with the ENS.

The Villani-Torossian Commission's report (see 1.3) has also pointed this lack of in-service training in France, and proposed several measures, some of which are inspired by both IREM groups and lesson-studies ([**97**]), and rely on the IREM network (see measures 15-16-27-28). Unfortunately, these new types of training courses, opened with constant resources despite the recommendations of the CNESCO (National Centre for the Study of the School System, [**29**]), only replace the training offered by the IREMs.

2.2. The publishing policy of the IREM network. Since their creation in 1969, the IREM have had an active publishing policy. It was unusual then, for teachers, to be encouraged to participate in writing activities related to their teaching. One of the original features of the IREM was to involve them in working groups, accompanied by university faculties, familiar with the scientific writing, with the aim of publishing booklets or articles resulting from these activities, for the other teachers. The written works mass thus collected in the IREM is considerable. They had a formative role when they were published and are a documentary source of great interest.

IREM publications are aimed at mathematics teachers (first and second degree, general or vocational education, higher education), teacher trainers, researchers in didactics, history or epistemology of mathematics. They are used by students in their initial professional training or for their initiation to research. They are of interest to anyone concerned with pedagogy or educational research. The themes addressed are diverse and reflect the wide variety of IREM missions on mathematics education from

[9] `https://www.fun-mooc.fr/courses/course-v1:ENSDeLyon+14003+session05/about`

kindergarten science to university: reports on classroom activities, history of mathematics, popularization of mathematics, connections with other disciplines...

Here is a classification of IREM productions, with information on their number for 2019-2020:

- "Booklets" (15 in 2018) from 50 to 100 pages; used to publish the work of a group in an IREM, as among those of the IREM&S of Poitiers, the booklets collection *Teaching mathematics using quantities* for cycle 3 or 4 (10 to 12 or 12 to 15 years old)([**67**],[**66**]). Booklets are also used to publish CII's work, e.g. [**31**] or [**35**];
- Books (11 in 2019-2020): used to publish the work of a CII around a work theme that it has set for itself ([**13**]), or to publish the proceedings of symposia ([**32**]);
- Articles (35 in 2019-2020), some published in the network reviews (see below), or in the reviews of CFEM partners (see 1.3), are used to synthesize the production of a working group or some of its members on all or part of their work.
- Some 50 "Working Documents" per year, often in digital form and available on-line on an IREM website; their contents reflect the work evolution within a group; they are a training to writing and serve as a pre-publication.

These works are referenced by the CII *Publimath* and its search engine, common to IREM and APMEP, that establishes for each production a form that provides information on editorial requirements and content of the document. It provides a link to a downloadable version of the document, and place it in the IREM digital library, if possible. *Publimath* references also publications reported as being useful for the teaching of mathematics in the French-speaking world. Among the 32 800 reading sheets filled by *Publimath*, 9 020 come from an IREM, including about 3 000 booklets or books: about 600 for elementary education, 1 000 for junior high school, 1 000 for high school. 10 000 pdf are placed in the IREM digital library. They also include about 100 videos, chapters in conference proceedings, journals articles and various texts available on the websites of different IREM. On average, since its creation, each IREM has published 90 booklets or books, 155 journal articles and 300 other resources (classroom situations, online courses, videos...).

Some previous resources have been revisited recently: either to rework them with the Mathematics General Inspectorate to make them official resources ([**100**], [**101**], [**102**], [**103**]); or to honour them for the IREM's 50th anniversary by publishing weekly short links between an old and a new production, or with regard to current events in mathematics education ([**58**]); to make a set of coherent selected resources for a teaching cycle [**36**]; or to increase the associated digital resources, [**34**],[**107**]. In addition, software or companion digital resources, or complex on-line resources are published (7 for the only CII TICE in 2019-2020, or the former ones for the CII Epistemology and

History) and also video resources (for example those of the IREM in Paris[10], increasingly provided in collections, for all the network. At last several theses are defended each year by members of the network, fed by the works of the groups.

The older network publications have been digitized and are available either in paper or in digital form for free online access on the IREM, the network or the CII websites, or via *Publimath*. The "booklets" and books are edited by an IREM itself, as part of a collection within a university press department (those of Franche-Comté (PUFC) or Limoges (PULIM)) or by private publishers (CII books for wider distribution).

The 4 journals of the IREM network below have national recognition by the High Council for Evaluation of Research and Higher Education[11] (HCERES) which has classified the first 3 in the "interfaces" category, and the last in the "research journals" category. They are freely accessible from their websites within two to three years following publication. They are pre-service and in-service teachers training implements, as well as trainers and researchers implements.

- *Repères IREM*[12], created in 1990, 122 issues 4 per year, 1178 articles published (on 1/05/2021). Its purpose is to serve as an interface between the research community, at a national level or in French-speaking countries. It informs the actors of the mathematical educational environment, but also of the related disciplines, about the work and reflection carried out jointly between teachers and researchers, in the classroom or in teacher training (of the first, second or higher degree). It focuses on current issues that affect the teaching communities: investigative approaches, interdisciplinarity, taking into account the pedagogical aspects of disability, assessment by competence, etc…, whether they relate to major debates or more simply to practical applications.
- *Petit x*[13], created in 1983, 113 issues $\simeq$ 3 per year, 4 articles per issue, 650 articles (on 1/05/2021); published by the IREM of Grenoble, sponsored by ARDM (see Ch.IV) and ADIREM. It is a review of mathematics didactics and of teaching practices analysis, at the secondary school level or at transitions (primary/junior, junior/high school, secondary/post-baccalaureate). It disseminates research, reflections, analyses and reports of work and class activities. The published articles contribute to ensure that this research and teaching practices feed into each other. *Petit x* also pursues international exchanges in the French-speaking world in the field of mathematics education, with the help of the foreign members of its editorial committee. It encourages young French-speaking researchers to present articles, of which a significant number are published.

[10] `https://irem.univ-paris-diderot.fr/videos-de-lirem-de-paris`
[11] `www.hceres.fr`
[12] `http://www.univ-irem.fr/spip.php?rubrique24`
[13] `https://irem.univ-grenoble-alpes.fr/revues/petit-x/`

- *Grand N*[14], created in 1973, 107 issues $\simeq 2$ per year, around 4 articles per issue (750 articles on 1/05/2021) and special sections; published by the IREM of Grenoble, supported by the COPIRELEM (see 1.2). Initially dedicated to mathematics teaching in primary school, it has been enriched since 1990 by the contribution of other scientific disciplines, and is the only French journal specifically dedicated to the sciences in elementary education. Its reading committee represents all the training bodies of primary school teachers. This review is at the interface between the field of research and that of professional practices. Its contents, anchored into research on mathematics and science teaching and in teacher training, aim to make it a real implement at the service of teachers and trainers, by broadening their field of possibilities in terms of classroom situations and by studies on the transition from primary school to *college*, as well as by situations that can be transferred to *college*.
- Les *Annales de Didactique et de Sciences Cognitives*[15], created in 1988, 25 issues, 6 to 10 articles per issue, 264 articles on 1/05/2021. It is an annual review of the IREM in Strasbourg. It publishes research results suitable to develop and stimulate reflection on mathematics education for all types of audiences (school children, high school students, students and adults in training): research concerning the pre-service and in-service training of teachers; on teaching in various socio-cultural contexts; combining a didactic theoretical framework and experimentation in the context of teaching; syntheses of research conducted in a particular field. The theoretical reference domains come from mathematics education but can also be based on cognitive psychology and linguistics.

Some IREM also publish or published a local journal, like *Feuilles de vigne* ("Vine Leaves", IREM of Dijon, 130 numbers, until 2014) or *L'ouvert* ("The Open", IREM of Strasbourg, until 2010, 118 numbers, 606 articles). Finally, the GREMA (Reflection Group on Mathematics Education in Africa, see 2.7 below) of the IREM of Paris sends its letter[16] which has become the journal of the International CII.

In addition, many teaching manuals take advantage, with or without adaptation, of the activities developed in the IREM, and the production of the network appears in partner journals[17]: *MathemaTICE*, published by *Sésamath*, *Recherches en Didactique des Mathématiques* by ARDM, *Bulletin Vert* and *Au Fil des Maths* by APMEP, *Educmath*, by Ifé.

2.3. Link to didactics, epistemology and history researches. The research in mathematics didactics has emerged in France in the IREM, which have profoundly influenced its development. The work that Guy Brousseau (Felix Klein medal in 2003)

[14] https://irem.univ-grenoble-alpes.fr/revues/grand-n/

[15] http://mathinfo.unistra.fr/IREM/publications/ADSC/#c62294

[16] https://irem.univ-paris-diderot.fr/la-lettre-de-grema

[17] http://revue.sesamath.net/ - http://rdm.penseesauvage.com/ - http://www.apmep.fr/-Le-Bulletin-Vert - https://afdm.apmep.fr/ - http://educmath.ens-lyon.fr/Educmath

has carried out for several decades at COREM (Observation and Research Center on Mathematics Education), created by the IREM of Bordeaux at his initiative, is a particularly symbolic illustration of this[18]. These works have nurtured the development of the Theory of Didactic Situations. Similarly, those of Régine Douady, at the IREM of Paris, have developed the Dialectic of the Tool-Object and the Frame Games, those of Yves Chevallard at the IREM of Marseille, the Theory of Didactic Transposition, those of Raymond Duval at the IREM of Strasbourg, his Semiotic Theory, and those of Michèle Artigue (Felix Klein medal in 2013) at the IREM of Paris, Jean-Baptiste Lagrange at the IREM of Rennes and Luc Trouche at the IREM of Montpellier, the Instrumental Approach to Technological Integration. Régine Douady has been president of ADIREM and Michèle Artigue president of IREM Scientific Committee.

The IREM have had a profound influence on the didactic research conducted in France through their operating methods and values. They allow the researchers in mathematics education to stay in close contact with the mathematical community and nurture the recognized epistemological sensitivity of their research. They provide them with an effective contact with the field of teaching, that of the classroom, which reflects in their problems, theoretical constructions, as well as in the methodological importance given to the didactic engineering.

Even when the institutionalization of didactic research led to the creation of specific research laboratories, in some universities, outside the IREM structure, these laboratories have kept close links with the IREM. And when the universitarization of pre-service teacher training (see 2.4 and Ch. III) led many didacticians to migrate to the new training institutes, the links with the IREM remained strong. This is attested by the durability of the commissions COPIRELEM and CORFEM (see 1.2) both dedicated to the pre-service training. More generally, the National Didactics Seminar, the CII Didactics and the organization of researchers in didactics around the ARDM (see 1.3) have always worked very closely. This impact on research is extended by other institutions (such as the French Institut for Education - Ifé) whose members may also work in some IREM and some LéA (Associated Education Places) can be IREM groups (Montpellier from 2015 to 2018, Grenoble since 2021).

The IREM have also promoted a collaborative and non-hierarchical vision of work between teachers and researchers. Their existence has allowed a percolation of research work among teachers that probably does not have an equivalent in any other discipline in France. The involvement of the IREM in in-service training is obviously essential. As is the work of applied research and of transposition of more fundamental research findings, carried out within IREM groups and CII, and irrigating the training sessions they offer, and consequently the pre-service training of mathematics teachers (see 2.4).

Since their creation, the IREM have also taken into account the teachers need to acquire a mathematics historical culture, which they often lacked, and to be provided with aids and resources to benefit from this culture. Specific working groups on these issues have thus been set up in many IREM. The mathematician and historian Jean-Luc Verley (IREM of Paris), from the beginning, wanted to propose to teachers and

[18]see interview `http://www.cfem.asso.fr/cfem/ICME-13-didactique-francaise`

students the study of original mathematical texts; Jean Dhombres (IREM of Nantes, once director of this IREM and president of the CS-IREM, was concerned to support multidisciplinary approaches and an epistemological reflection through historical work.

Thus, a specific research developed within the IREM, to support the introduction of a historical perspective into the teaching of mathematics from schools to university. From 1975, under the responsibility of Evelyne Barbin (IREM of Nantes) and Jean-Louis Ovaert, the CII "Epistemology and History of Mathematics" coordinated this work, quickly and durably becoming one of the most important commissions of the network. It still is today. It strives to disseminate the historical and epistemological works of the IREM to the mathematics teachers, but also to physical sciences and philosophy teachers, as well as to a wider audience of students and amateurs of mathematics, through the conferences organized every two years on a specific theme. The summer universities initiated in 1984 by the commission became European summer universities, that alternate with the conferences of the History and Pedagogy of Mathematics (HPM) group affiliated with ICMI. Evelyne Barbin chaired the HPM group from 2008 to 2012. Thomas Hausberger and Anne Cortella organized the HPM 2016 conference at the IREM of Montpellier.

The work of the CII Epistemology and History, and more broadly of the IREM in this field, has resulted in numerous local and national publications: monographs on thematic topics of interest in mathematics education or grouping together exponents of the integration of their history into teaching, collections of ancient texts commentaries... In total, the CII itself has published some thirty books, edited by the IREM, the Institut National de Recherche Pédagogique (now Ifé), university presses or private publishers. These works are internationally recognized, thanks in particular to the active participation of the members of the CII in European and HPM summer universities and symposia.

The collective book *Passerelles: enseigner les mathématiques par leur histoire en Cycle 3* (teaching mathematics by their history, [**107**]) received in 2019 the prize of the sciences teaching book of the French Sciences Academy. The CII also edited the book *Let history into the classroom* ([**13**]) to broaden its audience internationally.

2.4. Impact on pre-service training. In-service teacher training has been the primary mission of the IREM, but they have also been involved in the pre-service training of future primary or secondary school teachers. In 1990, when this training was restructured in autonomous university institutes (IUFM), the IREM were even offered to become their research laboratories. However they wanted to remain inside the universities, as close as possible to the mathematicians, in order to maintain their founding principle of close interaction with mathematics research. Despite this, the IUFM mainly recruited their mathematics trainers among the IREM facilitators. Thus they were able to influence the pre-service training of teachers, some of which came in turn, once professional, to work in an IREM. This symbiosis has been facilitated when, in 2006, the IUFM were integrated into a university, then in 2013, were replaced by the ESPE, and finally in 2019 became INSPE (see Ch. III), with a growing weight of research in initial training. A great attention is paid to the durability of IREM's involvement in the

new organisation of the studies of future teachers; ADIREM is working on this with the National Network of these structures. Some IREM are currently part of an INSPE (Lorraine and Picardie), and some other INSPE make part of the working time of their teachers or teacher-researchers available to the neighbouring IREM (Paris, Montpellier...).

Research, analysis of inventory of conditions, experimentation and critical reflection on the training of first and second degree teacher pre-service trainers are carried out in the IREM network by the commissions COPIRELEM, created as early as 1975 for the first degree, and the CORFEM created in 1993 for the second degree. Each of these CII organizes an annual "trainers' training" congress, hosted by an IREM. These congresses are currently part of the National Teacher Training Plan, and train the INSPE trainers.

An essential role of the IREM for future teachers is to convince them of the need to deepen their culture and professional practice throughout their careers; they facilitate their access to documentary resources (websites, libraries, etc.), and introduce them to use these tools and more generally offer them a framework adapted to the alliance of research and professional activity.

2.5. Impact on curricula. Since their creation, the IREM have been associated with the curricular reforms that have been successively carried out (see Ch. II). This association has taken various and complementary forms, including:

- the driving role of the network in the creation and animation, from the 80's, of the successive national reflection commissions for long term thinking of mathematics education evolution. The best known is the CREM also called Kahane Commission ([**88**]);
- the regular participation of IREM facilitators, from secondary and higher education, in expert groups in charge of drafting mathematics curricula, or even the management of these groups, and the opinions systematically transmitted by the network on developed projects (see 1.3);
- the privileged interaction of the National Education administration with certain CII: the CII high school and vocational high school within an interdisciplinary scientific group for the ongoing reform; the COPIRELEM for the current implementation of university training for future teachers prior to their professional training;
- the experimentation of the curricula projects entrusted to the IREM: the series of the *Suivis Scientifiques* were accessible to teachers as soon as the *collège* programs were set up in 1985;
- the systematic implementation of training and the resource documents production to support curriculum development or changes in the functioning of the school system: by the CII Probability for high school statistics in 2000 and probabilities in 2009; recently with IREM's productions reworked along with the Inspectorate (see 1.3); by the multidisciplinary groups of the IREM for the gradual introduction since 2009 of interdisciplinary work in secondary

school; currently, by the IREM for the creation of high school "labomaths", and for the RMC's training (1.3);
- vigilant attention to the evaluation of the curricula consequences and of the educational environment, by supporting EVAPM evaluation observatory of mathematics programs of APMEP[19].

The pioneering work of the IREM has often directly influenced curriculum reforms: in 1981, the profound evolution of analysis teaching was supported by the work of the CII *Analyse*; for the current introduction of historical perspectives in math teaching by the CII Epistemology and History (2.3); for technological integration with the participation in the development of many math software programs (*Cabri-géomètre, Géoplan* and Géospace, then Dgpad, or Xcas). Many experimental and research works are carried out in the IREM on numerical technologies, coordinated nationally by the CII TICE.

Since 2009, IREM has been supporting the introduction of algorithms in mathematics programs: in *lycée* (high school), then in elementary school and *collège* (junior high school, 2016) (Ch. II). Many groups have been created on this theme, working with technology teachers, that have offered training (including University Diplomas: at Marseille and Grenoble). This led to create an algorithmics group in the CII *lycée*. Then in 2017 the creation of a CII *Informatique* (or CIII), anticipating the introduction in 2019 of new computer science courses in *lycée*, and the announcement for 2020 of a new status of computer science teacher (with a CAPES for *NSI*, see Ch. II, section 3).

2.6. Actions to popularize mathematics. In addition to their main activity of teacher training, the IREM have gradually developed popularization activities, towards both students and the general public. They often serve as a dissemination channel for mathematics research laboratories.

Towards the students, the well-known involvement of the IREM is through the regional mathematics rallies (23 in 20 IREM), involving several hundred classes and more than 51 000 students. Some even exceeded borders: the Transalpin Rally, carried by the IREM of Franche-Comté, involving more than 4000 classes in 2018, from level 3 to 10, from Italy, French-speaking Switzerland, Belgium and Luxembourg. Teachers' meetings during rallies are also an opportunity to develop themes for in-service training. The problems proposed during these rallies and their analysis led to the publication of the "Panoramath" series, coordinated by the CII Pop'Math (7th volume in 2019, [**33**]).

The IREM cooperate with teachers and associations in actions to give students another vision of mathematics, encouraging scientific studies among girls and in modest social environments: *Math Kangaroo*[20], the CIJM games fair[21], MATh.en.JEANS workshops and congresses or Girls & Math Actions or days (see Ch. VI, sec. 5). The

[19] `https://www.apmep.fr/-Observatoire-EVAPM-`

[20] `http://www.mathkang.org/default.html`

[21] International Mathematics Games Comitee - `https://www.cijm.org/`

IREM offer Hippocampus or MathC2+ internships[22], that consist of reception of students inside university mathematics institutes, for several consecutive days to initiate them to research, by researchers, with or outside the classroom: students work on problems, experiment, debate and present their work as researchers. Initiated in biology by INSERM (National Institute of Health and Medical Research), the Hippocampus internships (see Ch. VI, sec. 3) have been adapted to mathematics since 2005 by the IREM of Aix-Marseille (15 internships per year) and now in Brest, Lyon, Toulouse...

The IREM participate in annual national actions for the dissemination of mathematics and its applications: "Sciences fair" (scientific activities fortnight for the general public and schools), "Mathematics Week" (under the auspices of the Ministry of National Education-MEN). In 2013, "Year of Mathematics for the Planet Earth", the IREM produced resources around this theme and in 2015 "Living Mathematics" forums (see ch. I sec. 5.1) were organized in Paris, Lyon and Marseille, relying on those three IREM. Other forums were held in 2017 in Lille, Rennes and Lyon. These forums mobilized several hundred teachers and researchers and welcomed many visitors. The network participates in 2019-2020 in the steering committee for the Mathematics Year (see ch. I, section 5.2).

Exhibitions have also been created in the network, attracting teachers with their sctudents and the general public: the exhibition *Regards sur les mathématiques - Itinéraires méditerranéens*[23] (IREM of Aix-Marseille), was translated to English for HPM 2016; the IREM of Grenoble is in charge of the animations and exhibitions of the museum *La grange des maths*[24]; the IREM of Montpellier is broadcasting the exhibition "Why mathematics?" created under the auspices of UNESCO with the participation of ICMI.

Finally, the network is strongly associated with other actors in scientific mediation and the promotion of scientific culture[25]. The consortium *Animath* participated in the CII Pop'math, as well as the society *Plaisir Maths*. Several working groups have been in common since 2017 between IREM and the *CultureMath* site of the ENS Paris and General Inspectorate, for teachers culture. Some members of the editorial committee of *Image des Maths* (CNRS website for the general public) come from the network (see ch. VI, section 4).

2.7. The international network. Since the 70', IREM have collaborated with universities abroad to develop teacher training in order to enhance structures based on their model: first in French-speaking Africa, with an IREM in Madagascar, then in Dakar (Senegal), that became IREMPT[26] (Mathematics, Physics and Technology), or in

[22] http://www.irem.univ-mrs.fr/Hippocampe - http://eduscol.education.fr/pid23341-cid54958/mathc2.html

[23] http://www.irem.univ-mrs.fr/expo2013/english.html

[24] https://www.echosciences-grenoble.fr/articles/la-grange-des-maths

[25] https://www.animath.fr/ - http://https://www.plaisir-maths.fr/ - https://www.cijm.org/ - http://culturemath.ens.fr/ - http://images.math.cnrs.fr/

[26] institut for research on mathematics, physics and technologies education – https://irempt.ucad.sn/

Niamey (Niger). Currently, IREM-type structures are still opening up: the IREM/UPC in Kinshasa (Democratic Republic of Congo) in 2014, with the support of the GREMA (IREM of Paris). Others were born in Latin America: IREM of Lima[27] (Peru), linked with the IREM of Caen, which organized its 9th international symposium on mathematics education (2018).

The network is developing to promote trainers and researchers exchanges and the creation of collective resources. The PReNum-AC project[28] (2012-2015) aimed at training teachers to use teaching technologies, online tools, and mathematics didactics. It involved trainees and trainers from Cameroon and the Republic of Congo, and produced many resources for the 12 grade. PReNum-AC micro (2017-2018) followed it for the expedition of micro-servers for access to students without Internet connection to online exercise databases, PReNum-AC resources and software for the class.

Two congresses were held to federate activities in IREM or similar structures. In 2016, the international conference "Training of mathematics teachers, here and elsewhere" was held in Strasbourg, with 80 participants from more than ten countries. It marked the creation of the IREM international network and led to an international mailing list and a teleseminar. GREMA's letter has become the international network's letter. Algeria has since been involved creating IREM. Another meeting was held in 2018, on the margins of the EMF2018 conference. Current events, projects, collaborations and difficulties where prensented for Ivory Coast, Senegal, Madagascar, Algeria, Democratic Republic of Congo and Brazil. In Hungary, an IREM group has been created. It mirrors another at the IREM of Paris Nord, supported by the Hungarian Academy of Sciences. Joint seminars allow the facilitators of the two groups to meet and develop collaborations.

The exchanges of resources, and even of trainers, between IREM and equivalent structures abroad encourages the popularization of mathematics in these countries: hippocampus internships (see 2.6) are developped in the IMPA-CNRS International Joint Unit in Rio of Janeiro and in the University of Roma[29].

2.8. Openness to other sciences. Although recognized at the national and international level, no structures comparable to the IREM have been created for other sciences in France. It was rather decided locally to entrust IREM with tasks, particularly on STEM, involving a close relationship between universities and the school world (valorization, professional development actions, high school/university link).

If the development of groups on the history and philosophy of mathematics, and therefore also of sciences, was natural, it is mainly because of changes in mathematics curricula and from links, that became necessary, with teachers from other sciences, that multidisciplinary scientific groups have been created in the IREM (see 2.5). They supported "interdisciplinary work in personal research" (in high school, then in junior high

[27] `http://irem.pucp.edu.pe/`

[28] Production of Digital Resources for Secondary Math Teaching in Central Africa – `http://prenumac.free.fr/`

[29] `http://www.matteoacclavio.com/HippocampeProject.html`

school, at last abandoned), as well as the introduction of algorithmics and then computer science in curricula. Following the example of the IREMPT in Dakar (2.7), some IREM have included sciences in their missions (e.g. with the creation of a multidisciplinary group for the initial training of science teachers in Montpellier), some became IRES (for Science Education): Toulouse and Orléans in 2015.

In 2016, the Ministry of National Education provided the network with additional resources to set up research groups in other sciences, in particular to overcome the difficulties of science education in high school resulting from the 2009-2011 programs. There are now (in 2019-2020) 36 secondary teachers and 39 researchers, in physics or chemistry, involved in 22 groups, 23 teachers ans 15 researchers in biology or geology in 9 groups, and 22 teachers and 53 researchers in computer science in 25 groups. Several Universities are considering asking their IREM to become IRES or IREM&S or IREMI (for Mathematics and computer science), and the change is recommanded by the Conference of Science Universities Deans.

In Poitiers, IREM became IREM&S in 2017; IREM of Montpellier will become IRES in 2021, IREM of La Réunion will become IREMI and an IREMI is to be born in Mayotte, linked to the former.

Particular attention is paid to keeping the IREM soul in this important evolution.

3. The people involved in the network

The functioning of the IREM network is not centralized. The presidents of ADIREM and CS-IREM (see 1.2), currently Anne Cortella, teacher-researcher in mathematics in Montpellier, and Christine Proust, researcher at CNRS in mathematics history, have important functions of management, coordination, dialogue with the authorities of ministries, academies and universities, and of representation of the network. But the network draws its strengths from its accessibility and the multiplicity and diversity of people organizing its activities: teachers at all levels, academics, researchers, teacher trainers. Their duties take many forms:

- organization of research groups, CII, trainings (more than 300 people);
- editorial, publishing or writing duties (Yves Ducel in Franche-Comté, Michèle Gandit in Grenoble, Mohamed Athlagh in Strasbourg...);
- digital information coordination or responsability for a server or search engine (Jérôme Germoni in Lyon, Hombeline Languereau in Franche-Comté, Vincent Paillet in Centre-Val de Loire, Jean-Louis Maltret in Marseille...);
- conferences and seminars organization (Christian Mercat for the international seminar in Lyon, 12 people for local seminars, local IREM or CII members for congresses);
- IREM direction, membership of boards (of an IREM, of the CS-IREM);
- organization of rallys, scientific facilitators (several dozen people)...

The number of these people can be estimated at several hundred at any given time, and therefore several thousand since the IREM network creation.

Despite the heavyness of tasks, often undertaken in addition to teaching or research, in spite of the material difficulties that have accumulated in recent years (reduction in

public subsidies, reduction in the resources allocated by the universities, reduction in the possibilities offered by the rectorates for the presence of teachers in groups and courses), it remains easy to find volunteers to suggest work themes or set up trainings, so strong is the conviction of the IREM's necessity and and so well rooted that their actions are carried out with great freedom.

VI Popularization and extracurricular activities

The activities of diffusion of mathematics aim to promote them to a large public, to democratize them, to inform about their usefulness, their beauty, their stakes, and to highlight the pleasure that one can have to practice them. These activities contribute to arouse vocations towards the different sciences by developing a more serene relationship with the discipline. They contribute to education and justify public investment in this field of mathematics. As far as extracurricular activities are concerned, the aim is both to stimulate young people who are passionate about mathematics and to change the attitude of those who are wary of it, showing them that "math is something other than what you think" by showing that, far from the constraints of the classroom and the weight of grades, doing mathematics has a playful dimension and brings great satisfaction. It is often observed that, placed in a situation that encourages and values his inventiveness, freed from the fear of "saying something stupid", integrated into a work team and assured that making mistakes is not bad, the student can truly unblock his attitude of passivity or rejection towards mathematics. The student is then able to take ownership of the subject. For those who already have a taste for mathematics, these extracurricular activities shall increase their motivation by helping them discover the wealth of mathematics and their applications.

While activities for the dissemination of mathematics, as well as science in general, had until the 1980s a limited impact, apart from the *Palais de la Découverte*, the situation has changed a lot since then, with a flourishing of various activities. They have recently received institutional recognition through the place given to them in the *Plan Mathématiques* implemented in 2018 by the Ministry of National Education following the 'Villani-Torossian' report [**149**].

The presentation that we make here, without being exhaustive, tries to give as complete a description as possible of the type of activities organized in France for the popularization of mathematics. Much of what is done in France is similar to what exists elsewhere. Nevertheless, there are some specificities that it is useful to underline in this introduction:

1) the desire not to reserve these activities for talented young people, but to address all audiences, particularly young people from disadvantaged backgrounds and girls;
2) the importance of projects in which the young people themselves are put in a research situation, within the framework of a collective work, often without a competitive dimension;
3) the presence of national structures allowing coordination and collaboration between the different initiatives.

1. Audiences

In principle, the dissemination actions concern the entire population, and in particular all school-age children. However, certain audiences are targeted more specifically:

- young people interested in mathematics and in particular, but not only, talented young people;
- young people from disadvantaged backgrounds or living away from university towns;
- girls.

2. Activities

2.1. Mathematics clubs, meetings, summer schools and courses. In many colleges and high schools (3500, i.e. about 25% of the total), and in some universities, clubs and workshops entirely or partially dedicated to mathematics operate. Among them, the MATh.en.JEANS workshops, clubs for the preparation of mathematical competitions, mathematical games clubs for the youngest, astronomy clubs...

Several types of courses and summer schools are organized during the vacations. The MathC2+ courses[1] are aimed at motivated students in the classes from *quatrième* to *première*[2] on a voluntary basis. They are particularly aimed at students who do not benefit from an environment conducive to the development of a long-term scientific study project and at girls. They allow students to discover mathematics in a different way, with a large part given to group work. The *Mat' les vacances* course, for *première* and *terminale* students, is aimed at the same public. Summer schools, some of which attract an international audience, are offered, as well as Olympic preparation courses mainly coordinated by the association "Animath".

Organized by the associations "Women and Mathematics" and "Animath", two other actions are aimed at an audience composed exclusively of girls. Each year, nearly 1500 secondary school girls from all over France take part to one of the fifteen or so "Girls, Mathematics and Computer Science: An Enlightening Equation" days: they attend a lecture given by a mathematician or a computer scientist, exchange views on scientific careers, take part in a play-forum to raise awareness of gender stereotypes, and meet women engaged in scientific careers during a speed-meeting (Section 5 gives more details). The *Rendez-vous des jeunes mathématiciennes et informaticiennes* (Meeting of Girls Mathematicians and Computer Scientists), which had seven occurrences in 2020, bring together about twenty high school girls for a program of lectures, meetings and discussions, and group work on open problems.

2.2. Scientific museums, exhibitions, forums, fairs. In addition to the *Palais de la Découverte* and the *Cité des Sciences*, located in Paris, there are many dedicated centers in the provinces, such as the *Maison des Mathématiques et de l'Informatique* (MMI) in Lyon, the *Maison de Fermat* (near Toulouse) or the *Grange des Maths* (south of Grenoble), and others in the making such as the *Maison Poincaré* (in Paris). Exhibitions, some of which are travelling, are proposed by the CCSTI (Scientific, technical and industrial culture centers)[3] and by associations or institutional structures. Forums

[1] `https://smf.emath.fr/la-smf/mathc2plus`

[2] See Figure 1 in chapter II.

[3] Centres de culture scientifique, technique et industrielle

and mathematical fairs are also proposed combining activities, demonstrations and exhibitions: *Salon de la culture et des jeux mathématiques* in Paris, Forum des mathématiques in Aix-en-Provence, *Jeux, Fête et Maths* in Eaubonne, Math'Gic in Gennevilliers, Festival *Les Maths dans tous leurs états* in Castanet-Tolosan, etc.

Created in 2000 on the occasion of the World Mathematical Year, the *Salon de la culture et des jeux mathématiques* has been held almost exclusively at Place Saint-Sulpice in Paris for twenty consecutive years, over 3 or 4 days in May. The pandemic has led to it being held virtually in 2020 and 2021, allowing it to reach a large and distant audience[4].

2.3. Public lectures and interventions of researchers in schools. Several cycles of conferences are organized: *Mathematic Park*; Mathematical evenings in Lyon; A text, a mathematician; An invention, mathematics; The *Cafés* of Statistics. Some research laboratories organize conferences in schools; researchers also give talks about manipulating mathematical objects (*Laboratoire de mathématiques d'Amiens*, *Labosaique* in Caen, *MathàLyon*, *Maths à Modeler* in Genoble, *Maison des Mathématiques de l'Ouest* in Nantes). Each year during the third week of March (including March 14), at the instigation of the Ministry of Education, the *Semaine des mathématiques* takes place with many actions in schools. Mathematics laboratories participate in national and international multi-scientific events such as the *Fête de la science* or the *Nuit des chercheurs*.

2.4. Publications, websites. In addition to general science magazines (*La Recherche*, *Pour la science*, *Sciences et avenir*, *Science et vie*, *Science et vie junior*, *Cosinus*), there are two magazines specifically dedicated to mathematics: *Tangente* and *Quadrature*. As for the sites: *Images des mathématiques*, proposed by the CNRS, is centered on the diffusion of research (see Section 4); *CultureMath* is more particularly addressed to teachers; *La Maison des Mathématiques de l'Ouest* (virtual) aims at federating the initiatives of diffusion in the Pays de Loire; *Florilège* tries to gather what touches the popularization of mathematics. Among the French-speaking YouTube channels: AuDiMath, Micmaths, *les 5 min Lebesgue*, Science4All, Chat Sceptique, El Jj.

2.5. Mathematical competitions in limited time, Olympiads. Several mathematical competitions are organized. They are open to all: the Mathador mental arithmetic competition (primary and secondary levels), the Kangaroo (all levels), mathematical rallies (organized regionally, mainly by team, at the primary and secondary levels), the Alkindi cryptography competition (classes of *quatrième*, *troisième* and *seconde*), and the national mathematics Olympiads (class of *première*). On selection: the General Mathematics Competition (*terminale*), the Animath Cup (end of secondary school), and in computer science: *Castor informatique* (the Computer Beaver) and Algoréa (all levels).

[4] `https://salon-math.fr`

2.6. Introduction to research. Several actions are carried out to put high school students in a research situation, including the Math.en.Jeans workshops, the TFJM² and the Hippocampe courses. Details are given in the following section.

2.7. Theater and visual arts. Several theater groups offer shows based on mathematics: the *Terraquée* and the *Ile logique* companies, which are both educational and entertaining. The shows offered by the *Comédie des Ondes* and *LAPS/équipe du matin* focus on inequalities and gender stereotypes. Exhibitions highlighting the artistic dimension of mathematical objects and concepts are offered, notably by the "European Society for Mathematics and Arts".

3. Examples of introductory research activities

3.1. MATh.en.JEANS. This association allows students from one or two twinned schools to work cooperatively during the year on a research topic proposed by a researcher, with a teacher as a reference. The two twinned schools meet 3 to 4 times during the year to compare their results and their research, in the presence of "their" researcher. In the spring, the students (together with their twins) present their results during one of the annual congresses (9 in France and 3 abroad), the high point of the program. These conferences are an opportunity for the students to discover the world of higher education. They attend two or three conferences given by mathematics researchers and then write a paper, which is included in the conference proceedings. Note that there is no competition, no prize: the reward is the participation in the congress and the presentation of the results obtained. During the last two years, despite the impossibility of gathering the students in these congresses, the MATh.en.JEANS workshops have been able to present their results during virtual conferences, which can be seen on the Youtube channel of the association: these videos constitute important resources for the teachers. These virtual congresses have brought together workshops from Hong Kong to San Francisco via Romania, Tunisia (and France). Moreover, in the context of the recent reform of the high school and the baccalaureate, the MATh.en.JEANS experience is a good preparation for the students for the *Grand oral* test.

3.2. TFJM². The French Tournament for Young Mathematicians exists since 2011, organized by Animath in collaboration with many higher education and research structures. This tournament is intended for high school students. In the framework of the TFJM², teams of six students are formed in high schools or sometimes by grouping several high schools. Open problems are proposed in January, on which these teams work for several months. They present their results by submitting them to the criticism of the other teams during regional tournaments (see details on the site `https://tfjm.org/`). The top teams compete in the National Tournament, and the top one or two teams compete in the International Tournament of Young Mathematicians (ITYM). A distance version of the ITYM², Mathematical Correspondences, was launched in 2018.

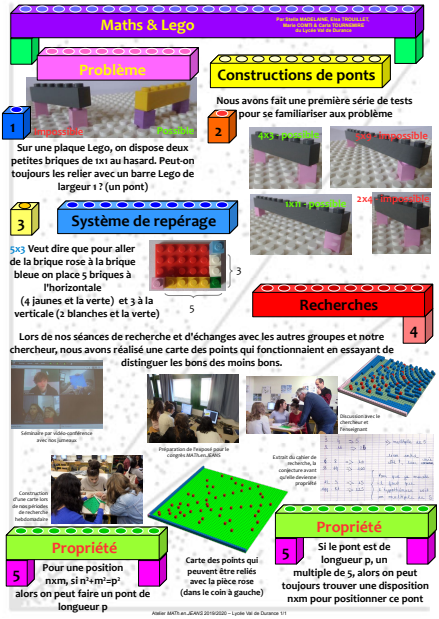

FIGURE 1. Poster during a MATh.en.JEANS workshop

3.3. Stages Hippocampe. The Hippocampe courses were invented by biologist researchers, to give high school students a research experience; they have been adapted to mathematics since 2005 at the IREM of Marseille; each year since 2006, there are between 12 and 18 courses in Marseille, more than 200 in total since the creation of the courses, with classes from many high schools and *colleges* between *quatrième* and *terminale* levels. This experience has since spread to other universities, in Brest, Nice, Amiens, Toulouse among others, as well as abroad.

During a Hippocampe course, a whole class is hosted at the university for 3 days, usually from Monday to Wednesday, and works on research problems related to a mathematical topic (for example, models of calculus, mathematical games, fractal objects, discrete mathematics, optimization, mathematics and music, infinity...); the students work in small groups for 2 and a half days, and present their results on a poster in the afternoon of the last day.

These internships, which were also done in the Second Chance School (a school for dropouts), give the students another view of mathematics, and allow them to work in conditions more similar to a research project. They also give high school teachers the opportunity to see another practice of the open problem.

FIGURE 2. Posters made during Hippocampe courses

One can find on the site `https://hippocampe.irem.univ-mrs.fr/` the (long) list of the subjects dealt with since 2010[5], a booklet showing posters made during these courses and, in the section "Communications and publications", some documents presenting these courses in more detail; we can point out in particular Clara Errico's master's thesis, which contains a complete bibliography on the subject and an analysis of the device within the framework of the anthropological theory of didactics.

4. A research dissemination site: *Images des mathématiques*

Images des mathématiques is an electronic journal born in 2009 with the ambition to present mathematical research and the profession of mathematician outside the scientific community. All articles are written by mathematical researchers and no article is written for researchers. After twelve years of existence and more than 3000 original articles, about 5000 visitors browse the site daily. And some of them participate actively in the life of the site, in particular by their work of rereading before the publication of the articles. *Images des mathématiques* is above all a great collective adventure, supported by the Audimath network of the CNRS and hosted by Mathrice[6], involving researchers, teacher-researchers, professors, students, hobbyists... and, more generally, all people who love mathematics.

One of the trademarks of *Images des mathématiques* is to give a prominent place to contemporary research. This is a challenge for the whole mathematical community, perhaps even more difficult than in other sciences because mathematics can quickly become very abstract and mathematicians are not generally used to explaining their work to the general public. The *Images des mathématiques* site aims at alleviating this problem by establishing a better communication between researchers and the public. It intends to shed light on the mathematical aspects of contemporary research and to highlight the historical, cultural and sociological dimensions.

4.1. Mathematical level of the articles. In order for the reader to choose his reading according to his mathematical level, the journal displays each article with a color:

- **Green track**: articles for the general public, with no particular knowledge of mathematics required.
- **Blue Track**: articles a little more difficult, but still readable without any problem if you remember a little bit your math classes at school.
- **Red track**: articles requiring a certain mathematical background, *a priori* at the level of a scientific high school diploma of today.
- **Black Track**: articles requiring more advanced knowledge, without being research level articles. They are in principle accessible to a student of scientific preparatory class.
- **Off-track**: rarer articles requiring even more specialized knowledge. They are in principle accessible to a bachelor-master level in math.

[5]See *stages de l'année en cours* as well as *archives des années précédentes*

[6]National portal of the CNRS for mathematics

4.2. Some of the sections of the site. *Images des mathématiques* is made up of a certain number of headings, here is a brief description of some of them.

- *Echoes of Research.* The articles, written by mathematicians for non-mathematicians, try to show what contemporary mathematical research looks like, from as many different points of view as possible.
- *The object of the month*. This section presents some of the most important, most beautiful, most useful, and even most surprising objects, ranging from the very concrete to the very abstract. Even if they are old, they are still relevant in research as (counter-)examples, as tools or as a source of inspiration.
- *The conjecture of the quarter.* Mathematics is full of open problems and conjectures, sometimes waiting for centuries for solutions or formal demonstrations. These challenges, which are also one of the driving forces of mathematical research, can and should be more widely known. The purpose of this section is to present one or more of them each quarter, with an emphasis on broad accessibility.
- *Images and visualization*. In this section, we present illustrations of mathematical situations and objects, some of which are at the heart of current research. We also give a place to artistic works using mathematical knowledge.
- *Out of school.* This section is intended for all those, young and old, who want to have fun while doing mathematics, around themes that are sometimes forgotten in school programs.
- *History of mathematics.* How do questions, concepts, results and theories take shape and change? How does mathematics fit into society and its time? What can we find in the writings of the past? Why have mathematicians and others become interested in the history and philosophy of their discipline over time? The section hosts articles on all of these questions.
- *Café des maths.* This section aims to establish a peer-to-peer relationship with the reader in order to talk about mathematics in an informal way, as if over a cup of coffee. It is a place where Internet users can come to react, exchange or launch a debate on a word or an expression that raises questions.
- *Mathematics, portraits.* This section aims to present men, women and places, all linked to mathematics and mathematical research, all different, naturally.
- *The press review.* The team of the press review concocts each month a rather exhaustive overview of the press. An immense work very much appreciated.
- *The comic book contest.* Every year, *Images des mathématiques* offers its readers a comic book contest and awards various prizes to the winners. The contest has become increasingly popular over the years.

The site *Images des mathématiques* is now well established in the landscape of mathematical dissemination, even if there is still a long way to go to reach a wider public, especially high school students and teachers.

5. The "Girls, Mathematics and Computer Science: An Enlightening Equation" Days

The associations Women and Mathematics and Animath organize the days "Girls, Mathematics and Computer Science: An Enlightening Equation" (in French *Filles, maths et informatique : une équation lumineuse*) with the support of several public and private institutions, including the Blaise Pascal Foundation.

The more math or computer science is involved in the studies, the fewer girls there are. We are convinced that it is not a matter of brains. And we put forward three main reasons: the omnipresence of social gender stereotypes, the lack of identification models and the lack of knowledge of the professions that these studies lead to. The aim of these days is to help participants identify gender stereotypes in order to try to free themselves from them, to discover accessible models of identification, and to learn more about the professions of mathematics, computer science and the digital sector. They

FIGURE 3. Announcement of a « Girls, Mathematics and Computer Science: An Enlightening Equation » day

are aimed at girls of secondary school, from classes between *troisième* and *terminale* levels, with a "scientific" flavour, and bring together up to 120 participants in a higher education establishment. Each year, we organize about 15 days throughout France and we want to increase this number so as to reach girls who are geographically distant and/or from disadvantaged social or cultural backgrounds.

The one-day program is structured around four key moments:

– a mathematics or computer science lecture by a young woman researcher or engineer,
– a workshop on gender stereotypes in relation to mathematics and computer science and/or the professions related to these fields, or a workshop on the professions of mathematics and computer science,
– a speed-meeting, meeting in small groups with women who have studied mathematics or computer science, or who work in these fields,

– a forum theater play: first performed by actors, it presents a high school girl at the time of orientation choices and grappling with the world around her, then the playwright establishes a critical dialogue on the characters' behaviors and offers girls to replay certain scenes.

These days are reserved for the girls in order to give them the possibility to speak easily and to think calmly about their choice of orientation and their future professional life, without having to play a role in front of the boys.

As an extension of the day, we offer the girls several documents: the *Zoom Métiers des mathématiques, de la statistique et de l'informatique* (Focus on Careers in Mathematics, Statistics and Computer Science), a document with resources on mathematics and computer science; as well as on career guidance; and a brochure for their parents. We suggest that they present what they did during the day to their peers, girls and boys, who did not attend. We also offer them the opportunity to be mentored by a female scientist for a year.

For more information, please visit the Women in Mathematics website:
`https://femmes-et-maths.fr/de-lecole-au-lycee/filles-et-maths-une-equation-lumineuse/`

6. Animath's international actions

Since 2010, the Animath association has been developing an international activity based on the creation and follow-up of extracurricular high school clubs, with the objective of motivating students to undertake higher scientific studies and to awaken their interest in future studies in France. The high school club, which has about equal numbers of girls and boys, meets regularly in small groups to solve "exotic" problems. It is supervised by local teachers and at a distance by French mathematicians familiar with the extracurricular activities. The spirit is that of a research seminar rather than a classroom. Each year the club organizes an intensive one-week session for a larger audience with the participation of Animath.

In Europe, it started in 2010 with the twinning of French and Romanian high school clubs leading to the participation of Romania in the ITYM team competition. This Franco-Romanian cooperation, extended to Moldova, was taken up, amplified and extended to more than ten high schools in five European countries by MATh.en.JEANS. This resulted in the very active ERASMUS+ Network "Math & Languages" with a strong component of comparison and enrichment of the teaching methods of each country. It should be noted that the actors of the project have written a reference guideook for all those who wish or will wish to set up a Maths&Languages workshop[7].

In Kosovo, the establishment in 2013 of a high school club and then a second club at the undergraduate level, has enabled the organization of well-attended summer sessions and has allowed several students to pursue mathematics studies up to the thesis in France, Germany or the USA. Animath has initiated the organization of a week of

[7]"Maths&languages guide. Mathematics research workshops held in a foreign language in secondary schools"

preparation for the International Olympiad common to the three Bulgarian, French and Romanian teams, in Craiova in 2018, in Pierrefonds in 2019.

In sub-Saharan Africa, in seven French-speaking countries, Animath coordinates thirty high school extracurricular math clubs. Started in Cameroon in 2011, and gradually extended to six other countries (Benin, Burkina Faso, Congo-Kinshasa, Congo-Brazzaville, Ivory Coast, Senegal), Animath's support for these clubs has been provided since 2018 under an agreement with Campus-France. Each year, one-week "mathematics forums" are held for high school students in the form of friendly introductory research courses. The focus is on Kinshasa, Senegal and Cameroon, where the partner is a non governmental organization of volunteer mathematicians, recognized by the Ministry[8]. In 2019, eight sessions in six countries brought together approximately 450 students and 100 of their teachers. At the end of each session, the students write their own evaluations, alone or in groups. These evaluations, which are often moving, demonstrate the profound change in their vision of mathematics.

In Senegal, Animath has signed an agreement with the private educational television station Télé-Ecole and since May 2020 has been broadcasting weekly mathematics programs: problems proposed from one week to the next, and popularizing videos from AuDiMaths. The regular audience is estimated at about 6000 high school students. A second agreement provides that the Cameroonian partner Promo-Maths, with the help of Animath, will produce extracurricular mathematics videos that will be transmitted for broadcast by Télé-Ecole. These programs, all free of rights, will be broadcast throughout French-speaking Africa.

And then came the coronavirus. The arrival of the pandemic abruptly broke the dynamics of implementation of extra-curricular activities. Especially since few African students have the necessary equipment and access to the Internet. And that few of their teachers have acquired the necessary experience. In spite of this, in Cameroon Promo-Maths was able to organize in 2020-21 both the Alkindi cryptography distance competition with 350 participants from 13 to 15 years old, and to present two teams - junior and senior - to the Francophone Mathematics Olympiad (OFM) implemented by Animath, with two bronze medallists among the seniors.

When travel from France and the meetings of the high school clubs become possible again, Animath will consider with its African partners the dissemination of the dematerialized initiatives implemented (Alkindi, OFM, television, ...) and the resumption of the patient implementation of the high school clubs, by valorizing the acquired experience of distance learning.

In China, since 2014, Animath has been in charge of the scientific part of the "Counting with the Other" contest, supported by the French Embassy and the MENJS, in collaboration with the China Education Association for International Exchange. The

[8]in Cameroon: Promo-Maths `www.promomaths.com/`; in Senegal, RC2S (*réseau des clubs scientifiques du Sénégal*) `https://www.facebook.com/rc2s.sn/?ref=pagew_internal`; in Congo-Kinshasa: SDI (*synergie pour le développement intégral du Congo*) `https://www.facebook.com/Synergie-Pour-le-Développement-Intégral-du-Congo-441310016031756/?ref=page_internal`

contest is aimed at students from the class of *seconde* in France and China and allows about 20 winners in each country to win a prize. Three editions have been held in 2014, 2017 and 2019. The tests are designed by a Franco-Chinese team, with each student composing in his/her own language. The construction of the competition's topics is an opportunity to exchange on the teaching of mathematics in France and in China.

In 2020, Animath participated in the CACIE-2020 (China Annual Conference for International Education and Expo). A session organized with representatives of Animath and MATh.en.JEANS, a French mathematics teacher in China and Chinese experts allowed discussions on experiences and methods to develop among young people the spirit of research and curiosity in mathematics, both in school and extracurricular fields. The next CACIE-2021 conference will take place in October 2021 in Beijing[9].

7. Actors of the popularization of mathematics

These activities are carried out by institutional structures (within universities or research organizations and in schools, museums and science centers), by learned societies and professional associations, by dedicated national or regional associations, by publishing houses... These structures maintain multiple relationships and frequently work together. In particular, we can mention:

- institutional, linked to the major research institutes: CNRS, INRIA (National Institute for Research in Digital Science and Technology), or to the Ministry in charge of National Education[10] and the Ministry of Higher Education: IREM, IHP, CIRM, MMI, Jacques Hadamard mathematics foundation (FMJH), mathematical sciences of Paris foundation (FSMP), Labex Lebesgue, Blaise Pascal foundation (FBP);
- learned societies and professional associations : SMF, SMAI, SFdS, APMEP, *femmes et mathématiques*;
- large national associations such as Animath, MATh.en.JEANS, Kangourou, *Fédération Française des Jeux Mathématiques* (FFJM), CIJM, or regional[11]: *Maths pour Tous*, *Science ouverte*, *Maths en Scène*, *Fermat Science*, *la Grange des maths*, *Plaisir Maths...*;
- publishers (dedicated to mathematics like Tangente, specialized in science or more generalist) and other commercial companies.

Some personalities also participate to embody the popularization of mathematics in France, with various actions (films, video channels, shows, tales, books...), one can think of Etienne Ghys, Mickaël Launay, Manu Houdart, Houria Lafrance, Marie Lhuissier, Olga Paris-Romaskevich,... it is impossible to quote them all.

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[9] `http://39.104.49.110/f/home/2019?langType=en`

[10] see Eduscol site, under the heading Mathematics and computer science `https://eduscol.education.fr/2045/culture-scientifique-technique-et-industrielle`

[11] `http://www.maths-pour-tous.org/v2/` – `https://scienceouverte.fr/` – `https://lesmathsenscene.fr/` – `https://www.fermat-science.com/` – `https://www.la-grange-des-maths.fr/` – `https://www.plaisir-maths.fr/`

Let's give some elements about the Blaise Pascal foundation[12] which was created in 2016. It is a national foundation, whose vocation is to promote, support, develop and perpetuate scientific mediation actions in mathematics and computer science for all citizens. Its actions are particularly focused on young women and socially and geographically disadvantaged students. For example, the days "Girls, Mathematics and Computer Science: An Enlightening Equation" organized by Animath and Women and mathematics described in section 5 have the support of the FBP. The foundation's ambition is to change the perception of mathematics and computer science by giving young people the desire to invest in this fundamental knowledge, and to put these disciplines back at the heart of the training of new generations in order to anticipate tomorrow's jobs.

Let us underline the important role of the IREMs which develop, in addition to their main activity of research and teacher training, an important activity of popularization of mathematics (cf. chapter V, section 2.6), mobilizing several hundreds of classes and several tens of thousands of students, sometimes from several countries, and aiming to give students and the general public another vision of mathematical activity, and to encourage scientific studies among girls and in modest social backgrounds.

Finally, a number of initiatives have been identified in the framework of the Year of Mathematics (chapter I, section 5.1), for which a guidebook has been published [**75**]. One will find other ideas, other contests, such as the comic strip contest *Bulles au carré*, organized by the on-line magazine *Images des mathématiques* (see section 4) and which had its 10th edition in 2021, or prizes such as the d'Alembert prize of the SMF, which aims at encouraging the diffusion of mathematical knowledge towards public, or the Tangente Prize for high school students. Concerning this prize of mathematical literature for high school students, let us mention the list Littéramath[13], fruit of a partnership between APMEP, Tangente, the IREM network and Publimath.

[12] `https://www.fondation-blaise-pascal.org/`

[13] `https://www.apmep.fr/-Litteramath`

Index

Bibliography

[1] Abdallah, E. (2021). Les mathématiques discrètes dans l'enseignement supérieur : une approche épistémologique et didactique. Thèse, Université de Reims (France) et Université Libanaise (Liban).

[2] Académie des sciences (Avis et recommandations, novembre 2010). La formation continue des professeurs enseignant les sciences à l'école, au collège, au lycée.

[3] Académie des sciences (rapport, mai 2013) : L'enseignement de l'information en France. Il est urgent de ne plus attendre. https://www.academie-sciences.fr/fr/Rapports-ouvrages-avis-et-recommandations-de-l-Academie/l-enseignement-de-l-informatique-en-france-il-est-urgent-de-ne-plus-attendre.html ou https://www.academie-sciences.fr/pdf/rapport/rads_0513.pdf

[4] Adler, J. (2000). Conceptualising resources as a theme for teacher education. Journal of Mathematics Teacher Education, 3, 205–224.

[5] Arnoux, P. (2018). Curriculum construction in France : Some remarks. In Y. Shimizu & R. Vithal (Eds.), School Mathematics Curriculum Reforms: Challenges, Changes and Opportunities. Proceedings of ICMI Study 24 Conference (pp. 491-499). Tsukuba : University of Tsukuba. https://drive.google.com/file/d/1xROQOjI8X_mJb5mmhH-G_OqoMaFE54Gy/view

[6] Aldon, G. Cusi, A., Schacht, F., Swidan, O. (2021). Teaching Mathematics in a Context of Lockdown: A Study Focused on Teachers'Praxeologies. Educ. Sci. 2021, 11(2), 38.

[7] Arsac, J. (1989). La didactique de l'informatique : un problème ouvert ? In Colloque francophone sur la didactique de l'informatique. Université René Descartes Paris (pp. 9–18).

[8] Artigue, M., Batanero, C., & Kent, P. (2007). Learning mathematics at post-secondary level. In F. Lester (ed.), Second Handbook of Research on Mathematics Teaching and Learning (pp.1011- 1049). Information Age Publishing, Inc., Greenwich, Connecticut.

[9] Artigue, M., Bosch, M., Chaachoua, H., Chellougui, F., Chesnais, A., Durand-Guerrier, V., Knipping, C., Maschietto, M., Romo-Vasquez, A;, & Trouche, L. (2019) The French Didactic tradition in mathematics. In W., Blum, M., Artigue, M. A. Mariotti, R. Sträßer, & M.Van den Heuvel-Panhuizen, M. European Traditions in Didactics of Mathematics. (pp.11-56) New York, NY: Springer.

[10] Artigue, M., Rinaldi, A.M. (2012). Design, curriculum et contrat social dans l'enseignement des mathématiques en France – Une étude de cas dans le cadre des tables rondes EMF2012 : Évolutions curriculaires récentes dans l'enseignement des mathématiques de l'espace francophone. In, Dorier J-L., Coutat S. (Eds), Enseignement des mathématiques et contrat social : enjeux et défis pour le 21e siècle – Actes du colloque EMF2012 (Plénières, pp. 24-52).

[11] Assude, T., Perez, J. M., Suau, G., Tambone, J., & Vérillon, A. (2014). Accessibilité didactique et dynamique topogénétique: une étude de cas. Recherches en didactique des mathématiques, 34(1), 33–57.

[12] Baily, C. & Finkelstein, N. D. (2010). Teaching and understanding of quantum interpretations in modern physics courses. Physical Review-Special Topics Physics Education Research, 6 (010101).

[13] Barbin, É., Guichard, J.-P., Moyon, M., Guyot, P., Morice-Singh, C., Métin, F., Bühler, M., Tournès, D., Chorlay, R., Hamon, G. (2018) Let history into the classroom. Springer, History of Mathematics Education. (http://www.univ-irem.fr/IMG/pdf/let_history_into_the_mathematics_classroom.pdf).

[14] Barrier, T., Durand-Guerrier, V. & Mesnil, Z. (2019). L'analyse logique comme outil pour les études didactiques en mathématique. Education et Didactique, 13-1, 61-81.

[15] Battie, V. (2007). Exploitation d'un outil épistémologique pour l'analyse des raisonnements d'élèves confrontés à la résolution de problèmes arithmétiques. Recherches en Didactique des Mathématiques, 27(1), 9-43.

[16] Battie, V. (2009). Proving in number theory at the transition from the secondary level to the tertiary level: between organizing and operative dimensions. In Lin F., Hsieh F.-J., Hanna G., & De Villiers

M. (Eds.), Proceedings of the ICMI Study 19 conference: Proof and Proving in Mathematics Education (pp. 71-76). The Department of Mathematics, National Taiwan Normal University Taipei, Taiwan.

[17] Bergé, A. (2016) Le rôle de la borne supérieure (ou supremum) dans l'apprentissage du système des nombres reels. In E. Nardi, C. Winsløw & T. Hausberger (Eds.), Proceedings of the First Conference of the International Network for Didactic Research in University Mathematics (INDRUM 2016, 31 March-2 April 2016) (pp. 33-42). Montpellier, France: University of Montpellier and INDRUM.

[18] Bilan de l'Année des mathématiques, Institut nationnal des sciences mathématiques et de leurs inreractions (2021). https://www.insmi.cnrs.fr/sites/institut_insmi/files/download-file/bilan-annee-mathsVF.pdf

[19] Bloch I., Gibel, P. (2019). A model to analyze the complexity of calculus knowledge at the beginning of University course – presentation and examples. Annales de didactique et de sciences cognitives, 24, 183-205.

[20] Bloch I., Gibel, P. (2016). A model to analyse the complexity of calculus knowledge at the beginning of University course. In E. Nardi, C. Winsløw & T. Hausberger (Eds.), Proceedings of the First Conference of the International Network for Didactic Research in University Mathematics (INDRUM 2016, 31 March-2 April 2016) (pp. 43-52). Montpellier, France: University of Montpellier and INDRUM.

[21] Bloch I., Gibel, P. (2011). Un modèle d'analyse des raisonnements dans les situations didactiques : étude des niveaux de preuves dans une situation d'enseignement de la notion de limite. Recherches en Didactique des Mathématiques, 31-2, 191-228.

[22] Bridoux, S., de Vleeschouwer, M., Grenier-Boley, N., Khanfour-Armalé, R., Lebrun, N., Mesnil, Z. & Nihoul, C. (2020). The professional identity of teacher-researchers in mathematics. In T. Hausberger, M. Bosch & F. Chelloughi (Eds.), Proceedings of the Third Conference of the International Network for Didactic Research in University Mathematics (INDRUM 2020, 12-19 September 2020) (pp. 219-228). Bizerte, Tunisia: University of Carthage and INDRUM.

[23] Burton, L. (2001). Research Mathematicians as Learners-and what mathematics education can learn from them. British Educational Research Journal, 27(5), 589-599.

[24] Cadrage des concours à partir de la session 2022
- CRPE : https://www.legifrance.gouv.fr/jorf/id/JORFTEXT000043075701
- CAPES : https://www.legifrance.gouv.fr/jorf/id/JORFTEXT000043075486
- CAPLP : https://www.legifrance.gouv.fr/jorf/id/JORFTEXT000043075622

[25] Cartier, L. (2008). Le graphe comme outil pour enseigner la preuve et la modélisation. Thèse. Université Grenoble 1.

[26] Chellougui, F. (2009). L'utilisation des quantificateurs universel et existentiel en première année d'université, entre l'explicite et l'implicite. Recherches en didactique des mathématiques, 29-2, 123-154.

[27] Chorlay, R. (2018). An empirical study of the understanding of formal propositions about sequences, with a focus on infinite limits. In V. Durand-Guerrier, R. Hochmuth, S. Goodchild & N.M Hogstad (Eds.), Proceedings of the Second Conference of the International Network for Didactic Research in University Mathematics (INDRUM 2018, 5-7 April 2018) (pp. 24-33). Kristiansand, Norway: University of Agder and INDRUM.

[28] Chorlay, R. (2019). A Pathway to a Student-Worded Definition of Limits at the Secondary-Tertiary Transition. International Journal of Research in Undergraduate Mathematics Education, 5 (3), 267–314.

[29] CNESCO (2020). La formation continue et le développement professionnel des personnels d'éducation. http://www.cnesco.fr/wp-content/uploads/2021/03/Cnesco_CCI_formation_continue_Dossier_de_synthese_210226.pdf

[30] Colmez, François, Cécile de Hosson, Joëlle Pichaud, Aline Robert, Hommage à André Revuz, L'engagement universitaire, l'héritage didactique. Laboratoire de didactique André Revuz (2010). hal-02345760

[31] Commission inter-IREM Collège (2016). Agrandir, réduire... dans tous les sens. APMEP https://www.univ-irem.fr/spip.php?article1461

[32] Commission inter-IREM Histoire et Épistémologie, Barbin, E. et Maltret, J.-L. (éditeurs) (2015). Les mathématiques méditerranéennes : d'une rive et de l'autre. Actes du colloque Inter-IREM Epistémologie et Histoire des mathématiques Marseille France 2013. https://www.univ-irem.fr/spip.php?article1167

[33] Commission inter-IREM Pop'Maths (2019). Panoramath'7. Collection Panoramath, Co-édition CIJM-IREM-APMEP.

[34] Commission inter-IREM TICE sous la direction de Deleuze, G. et Padilla, P.(2015). Créer avec GeoGebra - Exemples de réalisations et fiches techniques. IREM de Paris, ré-édité par Cassini. Ressource augmentée en ligne https://tice.univ-irem.fr/lexique/co/site.html

[35] Commission inter-IREM Université (2017). Limites de suites réelles et de fonctions numériques d'une variable réelle : constats, pistes pour les enseigner. IREM de Paris https://www.univ-irem.fr/IMG/pdf/version_finale.pdf

[36] COPIRELEM (depuis 2017). Malette maternelle : la construction du nombre. ARPEME, http://www.arpeme.fr/m2ep/

[37] Coppé, S. (2018). Évaluation et didactique des mathématiques : vers de nouvelles questions, de nouveaux travaux. Revue Mesure et Évaluation en Éducation. 41(1), 7-39.

[38] Cornu, B., & Ralston, A. (1992). The influence of computers and informatics on mathematics and its teaching (Vol. 44). Unesco.

[39] Cour des comptes (octobre 2017), Gérér les enseignants autrement. Une réforme qui reste à faire. Rapport public thématique

[40] d'Enfert, Renaud et Gispert, Hélène. Une réforme à l'épreuve des réalités : le cas des "mathématiques modernes" au tournant des années 1970, https://halshs.archives-ouvertes.fr/file/index/docid/536328/filename/Reforme_Math_1970.pdf.

[41] DeBellis, V.A, Rosenstein, J.G. (2004). Discrete mathematics and Proof in the High School. ZDM, 36(2,3), 44–84, 82–116.

[42] de Hosson, C. Décamp, N, Morand, E., & Robert, A. (2015). Approcher l'identité professionnelle d'enseignants universitaires de physique: un levier pour initier des changements de pratiques pédagogiques. Recherches en Didactique des Sciences et des Technologies, 11, 161-190.

[43] Deloustal-Jorrand, V. (2004). L'implication mathématique : étude épistémologique et didactique. Étude sous trois points de vue : raisonnement déductif, logique formelle et théorie des ensembles. Thèse. Université J. Fourier, Grenoble 1.

[44] Derouet, C. (2019). Co-construction d'une séquence d'enseignement articulant lois à densité et calcul intégral en terminale S : présentation d'une méthodologie de type ingénierie didactique collaborative. In S. Coppé, É. Roditi, V. Celi, F. Chellougui, F. Tempier, C. Allard,... M. Kiwan-Zacka (Eds.), Nouvelles perspectives en didactique : géométrie, évaluation des apprentissages mathématiques (pp. 487–494). Grenoble : La Pensée Sauvage.

[45] DIMACS. (2001). Center for Discrete Mathematics and Theoretical Computer Science: Educational Program. http://dimacs.rutgers.edu/Education

[46] Dorier, J.-L. (Ed.) (1997). L'algèbre linéaire en question, collection Bibliothèque de Recherches en Didactique des Mathématiques. Grenoble : La Pensée Sauvage.

[47] Dorier, J.-L. & Sierpinska, A. (2001). Research into the teaching and learning of linear algebra. In D. Hoton (ed.), The teaching and learning of mathematics at university level. Dordrecht: Kluwer.

[48] Durand-Guerrier, V. & Arsac, G. (2005). An epistemological and didactic study of a specific calculus reasoning rule. Educational Studies in Mathematics, 60(2), 149-172.

[49] Durand-Guerrier, V., Meyer, A. & Modeste, S. (2020). Didactical issues at the interface of mathematics and computer science. In: G. Hanna, M. De Villiers (eds), Proof Technology in Mathematics Research and Teaching, 115-138.

[50] Durand-Guerrier, V., Boero, P., Douek, N., Epp, S. & Tanguay, D. (2012). Examining the Role of Logic in Teaching Proof. In G. Hanna & M. de Villiers (eds), ICMI Study 19 Book: Proof and Proving in Mathematics Education (p. 369- 389). New-York : Springer.

[51] Durand-Guerrier, V., Hochmuth, R., Nardi, E. & Winsløw, C. (2021) (Eds.). Research and development in University Mathematics Education. Collection ERME Series, Routledge Editions.

[52] Durand-Guerrier, V., Kazima, M. ,Libbreht, P., Njomgang Ngansop, J.L., Salekhova, L.N. , Tuktamyshov, N., Winslow C. (2016). Challenges and Opportunities for Second Language Learners in Undergraduate Mathematics. In Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J., Planas, N., Phakeng, M., Valero, P., Villavicencio Ubillús, M. (Eds.), Mathematics Education and Language Diversity, The 21st ICMI Study (pp.85-101). Springer.

[53] Durand-Guerrier V., & Tanguay D. (2018). Working on proofs as contributing to conceptualization – The case of R Completeness. In Stylianides A.J., Harel G. (eds), Advances in Mathematics Education Research on Proof and Proving. An international perspective (pp.19-34). Springer.

[54] Durand-Guerrier, V., & Vivier, L. (2016). Densité de D, complétude de R et analyse réelle. Première approche. In E. Nardi, C. Winsløw & T. Hausberger (Eds.), Proceedings of the First Conference of the International Network for Didactic Research in University Mathematics (INDRUM 2016, 31 March-2 April 2016) (pp. 143-152). Montpellier, France: University of Montpellier and INDRUM.

[55] El Hage, S. & Ouvrier-Buffet, C. (2018). Les démarches de chercheurs en physique et en mathématiques. Enjeux didactiques d'une nouvelle approche épistémologique. Recherches en Éducation, 34, 106-126.

[56] Fluckiger, C. (2019). Une didactique de l'informatique scolaire. Presses Universitaires de Rennes.

[57] Gandit, M. (2009). Etude épistémologique et didactique de la preuve en mathématiques et de son enseignement. Une ingénierie de formation. Thèse, Université Joseph Fourier, Grenoble I.

[58] Gandit, M., Meninni, C.(2019. Brèves hebdomadaires des IREM (1 à 35). Portail des IREM. `http://www.univ-irem.fr/spip.php?rubrique513`

[59] Gaona, J. (2018). Elaboration d'une base d'exercices en ligne comme processus de formation des professeurs de mathématiques. Thèse de didactique des disciplines, spécialité didactique des mathématiques. Université Paris Diderot.

[60] Gardes, M.-L. (2013). Étude de processus de recherche de chercheurs, élèves et étudiants, engagés dans la recherche d'un problème non résolu en théorie des nombres. Thèse, Université Lyon 1.

[61] Gibel, P. (2020). Analyse en théorie des situations didactiques d'une ingénierie visant une première approche de la notion de limite finie d'une suite. Revue Québécoise De Didactique Des Mathématiques, 1, 153-189.

[62] Grapin, N. (2015). Étude de la validité de dispositifs d'évaluation et conception d'un modèle d'analyse multidimensionnelle des connaissances numériques des élèves de fin d'école, Thèse de Doctorat, Université Paris-Diderot.

[63] Grenier, D., & Payan, C. (1998). Spécificité de la preuve et de la modélisation en mathématiques discrètes. Recherches en didactique des mathématiques, 18(2), 59–100.

[64] Grenier-Boley, N. (2019). La recherche en mathématiques : une ressource pour les didacticiens ? Habilitation à Diriger des Recherches, Université Paris Diderot.

[65] Grenier-Boley, N., Nicolás, P., Strømskag, H. & Tabchi, T. (2021). Mathematics teaching practices at university level. In V. Durand-Guerrier, R. Hochmuth, E. Nardi & C. Winsløw (Eds.), Research and development in University Mathematics Education. Collection ERME Series, Routledge Editions.

[66] Groupe collège de l'IREM de Poitiers (depuis). Enseigner les mathématiques au cycle 4 à partir des grandeurs. (Collection) IREM de Poitiers. `http://irem2.univ-poitiers.fr/portail/index.php?option=com_content&view=article&id=180&Itemid=197`

[67] Groupe collège de l'IREM de Poitiers (depuis). Enseigner les mathématiques au cycle 3 à partir des grandeurs. (Collection) IREM de Poitiers. `http://irem2.univ-poitiers.fr/portail/index.php?option=com_content&view=article&id=180&Itemid=197`

[68] Gueudet, G. (2008) Investigating the secondary–tertiary transition. Educational Studies in Mathematics, 67 (3), 237-254.

[69] Gueudet, G., Bueno-Ravel, L., Modeste, S., & Trouche, L. (2017). Curriculum in France: a national frame in transition. In D. Thompson, M.A. Huntley, & C. Suurtamm, International Perspectives on Mathematics Curriculum (pp. 41-70). Charlotte, NC: International Age Publishing

[70] Gueudet, G., Buteau, C., Mesa, V., & Misfeldt, M. (2014). Instrumental and documentational approaches: from technology use to documentation systems in university mathematics education. Research in Mathematics Education, 16(2), 139-155.

[71] Gueudet, G., & Parra, V. (2017). Teachers' collective documentation work: a case study on tolerance intervals. In T. Dooley & G.Gueudet (Eds) Proceedings of the Tenth Congress of the European Mathematical Society for Research in Mathematics Education, (pp.3707-3715) Dublin, Ireland.

[72] Gueudet, G., Pepin, B., Sabra, H., & Trouche, L. (2016). Collective design of an e-textbook: teachers' collective documentation. Journal of Mathematics Teacher Education, 19(2-3), 187-203.

[73] Gueudet, G., & Thomas, M. (2018). The secondary-tertiary transition in mathematics. In S. Lerman (Ed.), Encyclopedia of Mathematics Education. Springer.

[74] Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for teachers? Educational Studies in Mathematics, 71(3), 199-218.

[75] Guide National Année des mathématiques 2019-2020 (MENJ, MESRI, CNRS) https://cache.media.eduscol.education.fr/file/CST/90/2/guide-national-maths_A5/_1183902.pdf

[76] Hache, C. & Mesnil, Z. (2015). Pratiques Langagières et preuves. Actes du 22e colloque de la COREM. Nîmes, juin 2015. Online : https://hal.archives-ouvertes.fr/hal-01285116v1

[77] Harel, G. (2000). Three principles of learning and teaching mathematics: Particular reference to linear algebra - Old and new observations. In J.-L. Dorier (Ed.), On the Teaching of Linear Algebra (pp.177-190). Kluwer Academic Publishers.

[78] Hart, E., & Sandefur, J. (2018). Teaching and Learning Discrete Mathematics in the School Curriculum Worldwide. An ICME-13 Monograph. Springer.

[79] Hausberger, T. (2017). La dialectique objets-structures comme cadre de référence pour une étude didactique du structuralisme algébrique. Education et Didactique, 11(2), 131-151.

[80] Hausberger, T. (2018). Structuralist Praxeologies as a Research Program on the Teaching and Learning of Abstract Algebra. International Journal of Research in Undergraduate Mathematics Education, 4(1), 74-93.

[81] Hausberger, T. (2018). Abstract Algebra Teaching and Learning. In S. Lerman (ed.), Encyclopedia of Mathematics Education. Springer.

[82] Heublein, U. (2014). Student Drop-out from German Higher Education Institutions. European Journal of Education, 49(4), 497–513.

[83] Horoks J. & Pilet J. (2015) Etudier et faire évoluer les pratiques d'évaluation des enseignants de mathématiques en algèbre au collège dans le cadre d'un Léa. In Theis, L. (Ed.),Pluralités culturelles et universalité des mathématiques : enjeux et perspectives pour leur enseignement et leur apprentissage, Actes du 6e colloque Espace Mathématique Francophone (EMF 2015 – GT9, 10-14 octobre 2015) (pp. 791–804). Alger, Algérie : Faculté de Mathématiques.

[84] Rapport IGAENR no 2015-024 (mai 2015). Bilan qualitatif des emplois d'avenir professeur.

[85] INDRUM (2018). https://hal.archives-ouvertes.fr/INDRUM/

[86] Jaworski, B. & Matthews, J. (2011). How we teach mathematics: discourses on/in university teaching. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), Proceedings of the Seventh Congress of the European Mathematical Society for Research in Mathematics Education (pp. 2022-2032). Rzeszów, Poland: University of Rzeszów and ERME.

[87] Jovignot, J. (2020). Etude de la transposition didactique du concept d'idéal : écologie des savoirs et problématique de l'entrée dans la pensée structuraliste, en France et en Suisse. Thèse, Université de Montpellier.

[88] Kahane, J.-P. (2002). Enseignement des sciences mathématiques : Commission de réflexion sur l'enseignement des mathématiques : Rapport au ministre de l'éducation nationale (CNDP). Odile Jacob. http://www.cfem.asso.fr/ressources/rapports-enseignement-mathematiques/commission-kahane

[89] Kuzniak, A., Montoya, E., Vandebrouck, F., & Vivier, L. (2015). Le travail mathématique en analyse de la fin du secondaire au début du supérieur : identification et construction, cours à la 18ième école

d'été de didactique des mathématiques, In Y. Matheron, G. Gueudet et al. (Ed.), Enjeux et débats en didactique des mathématiques (pp 47-66). Grenoble : La Pensée Sauvage.

[90] Kuzniak, A., Tanguay, D., & Elia, I. (2016). Mathematical Working Spaces in schooling: an introduction. ZDM Mathematics Education, 48, 721–737.

[91] Lalaude, M. (2016). L'enseignement de l'algèbre linéaire au niveau universitaire – Analyse didactique et épistémologique. Thèse de l'Université de Pau et des Pays de l'Adour.

[92] Lalaude, M., Gibel, P., Bloch, I., Lévi, L. (2018). A TDS analytical framework to study students' mathematical activity - An example: linear transformations at University. In V. Durand-Guerrier, R. Hochmuth, S. Goodchild & N.M. Hogstad (Eds), Proceedings of INDRUM 2018 – Second conference of the International Network for Didactic Research in University Mathematics (p. 234-243). University of Agder, Kristiansand: Norway.

[93] Lautesse, P., Vila Valls, A., Héraud, J.L. & Chabot, H. (2015). Teaching quantum physics in upper secondary school in France: 'quanton' versus 'wave-particle' duality, two approaches of the problem of reference. Science & Education, 24 (7), 937-955.

[94] Laval, D. (2018). L'algorithmique au lycée entre développement de savoirs spécifiques et usage dans différents domaines mathématiques. Thèse de l'Université Paris Diderot Paris 7, retrieved at `https://hal.archives-ouvertes.fr/tel-01943971v1`

[95] Lockwood, E., Ellis, A. & Lynch, A. (2016). Mathematicians' example-related activity when exploring and proving conjectures. International Journal of Research in Undergraduate Mathematics Education, 2(2), 165-196.

[96] Loisy, C., Sabra, H., Courtney, S. A., Rocha, K., Glasnović Gracin, D., Aldon, G., Front, M., Gardes, M.-L., Taranto, E., Arzarello,F. & Robutti, O. (2019). Analyzing Teachers' Work with Resources: Methodological Issues. In Trouche, L., Gueudet, G., Pepin, B. (eds), The "Resource" Approach to Mathematics Education. (p. 257-321). New York, NY: Springer.

[97] Masselin, Blandine & al (2020) Ingénierie de formation en mathématiques de l'école au lycée: des réalisations inspirées des Lesson Studies. Presses Universitaires de Rouen et du Havre. `https://hal-normandie-univ.archives-ouvertes.fr/hal-03198032`

[98] Mesa, V., & Griffiths, B. (2012). Textbook mediation of teaching: An example from tertiary mathematics instructors. Educational Studies in Mathematics, 79, 85-107.

[99] Mesnil, Z. (2014). La logique : d'un outil pour le langage et le raisonnement mathématiques vers un objet d'enseignement. Thèse de doctorat, Université Paris Diderot, Paris.

[100] Ministère de l'Éducation nationale, de l'Enseignement supérieur et de la Recherche et ADIREM (2016). Mathématiques et maîtrise de la langue. Eduscol, cycles 3 et 4, Mathématiques, Ressources transversales. `https://cache.media.eduscol.education.fr/file/Ressources_transversales/99/6/RA16_C3C4_MATH_math_maitr_lang_N.D_600996.pdf`

[101] Ministère de l'Éducation nationale, de l'Enseignement supérieur et de la Recherche et ADIREM (2016). Mathématiques et quotidien. Eduscol, cycles 3 et 4, Mathématiques, Ressources transversales. `https://cache.media.eduscol.education.fr/file/Ressources_transversales/99/8/RA16_C3_C4_MATH_math_et_quotidien_600998.pdf`

[102] Ministère de l'Éducation nationale, de l'Enseignement supérieur et de la Recherche et ADIREM (2016). Les mathématiques par les jeux. Eduscol, cycles 3 et 4, Mathématiques, Ressources transversales. `https://cache.media.eduscol.education.fr/file/Maths_par_le_jeu/92/4/01-RA16_C3_C4_MATH_math_jeu_641924.pdf`

[103] Ministère de l'Éducation nationale, de l'Enseignement supérieur et de la Recherche et ADIREM (2016). Mathématiques, monde économique et professionnel et parcours Avenir. Eduscol, cycles 3 et 4, Mathématiques, Ressources transversales. `https://cache.media.eduscol.education.fr/file/Ressources_transversales/99/6/RA16_C3C4_MATH_math_maitr_lang_N.D_600996.pdf`

[104] Modeste, S. (2012). Enseigner l'algorithme pour quoi ? Quelles nouvelles questions pour les mathématiques ? Quels apports pour l'apprentissage de la preuve ? Thèse de l'Université de Grenoble.

[105] Montoya Delgadillo E., Páez Murillo, R-E., Vandebrouck F., Vivier L. (2018) Deconstruction with Localization Perspective in the Learning of Analysis. International Journal of Research in Undergraduate Mathematics Education, 4(1), 139–160.

[106] Montoya Delgadillo, E. & Vivier, L. (2016). Mathematical Working Spaces and Paradigms as an analysis tool for the teaching and learning of analysis. ZDM, 48(6), 739-754.

[107] Moyon, M., & Tournès, D., Dir. (2018). (Passerelles : enseigner les mathématiques par leur histoire en Cycle 3. Association pour l'élaboration et la diffusion de ressources pédagogiques sur l'enseignement des mathématiques à l'école (ARPEME) Bouc Bel Air, 2018 Collection : Ressources et formation. `http://www.univ-irem.fr/spip.php?rubrique1597`. Site compagnon avec ressources complémentaires `http://www.univ-irem.fr/spip.php?rubrique505`.

[108] Nardi, N., Jaworski, B., & Hegedus, S. (2005). A Spectrum of Pedagogical Awareness for Undergraduate Mathematics: From tricks to techniques. Journal for Research in Mathematics, 36(4), 284-316.

[109] Note d'information de la DEPP. no 12.23. (2012). PACEM (Projet pour l'acquisition de compétences par les élèves en mathématiques) : une expérimentation de formation continue d'enseignants en mathématiques. Résultats en sixième.

[110] Note d'information de la DEPP, no 19.08. Mars 2019. L'évolution des performances en calcul des élèves de CM2 à trente ans d'intervalle (1987-2017).

[111] Note d'information de la DEPP, no 19.33. Sepembre 2019. Cedre 2007-2013-2018 – Sciences en fin de collège : des résultats en baisse.

[112] Note d'information de la DEPP, no 19.48. Novembre 2019. Choix de trois spécialités en première générale à la rentrée 2019 : 15 combinaisons pour 80 % des élèves.

[113] Note d'information de la DEPP, no 21.22 Mai 2021. Des choix de spécialités plus classiques en première comme en terminale pour les élèves d'origine sociale favorisée.

[114] Ouvrier-Buffet, C. (2009). Maths à Modeler: Research-Situations for Teaching Mathematics. In Barbeau, E. & Taylor, P. (Eds.), ICMI Study 16, Challenging Mathematics in and beyond the Classroom (pp. 23-29). Springer.

[115] Ouvrier-Buffet C. (2013). Modélisation de l'activité de définition en mathématiques et de sa dialectique avec la preuve – Étude épistémologique et enjeux didactiques. Note de synthèse Habilitation à Diriger des Recherches. Université Paris Diderot.

[116] Ouvrier-Buffet, C. (2015). Modéliser l'activité de définition : vers de nouvelles perspectives en didactique. Recherches en didactique des mathématiques, 35(3), 313-356.

[117] Ouvrier-Buffet, C., Meyer, A., & Modeste, S. (2018). Discrete mathematics at university level - Interfacing mathematics, computer science and arithmetic. In V. Durand-Guerrier, R. Hochmuth, S. Goodchild & N.M. Hogstad (Eds), Proceedings of INDRUM 2018 – Second conference of the International Network for Didactic Research in University Mathematics (p. 255-264). University of Agder, Kristiansand: Norway.

[118] Pepin, B., & Gueudet, G. (2020). Studying teacher collaboration with the documentational approach: from shared resource to common schemes? ICMI study 25, Teachers of mathematics working and learning in collaborative groups. Lisboa, Portugal.

[119] Pepin, B., Xu, B., Trouche, L., & Wang, C. (2017). Developing a deeper understanding of mathematics teaching expertise: Chinese mathematics teachers' resource systems as windows into their work and expertise. Educational studies in Mathematics, 94(3), 257–274, `http://rdcu.be/koXk`

[120] Peteers, F. (2020). Apports croisés de la didactique et de la cognition numérique pour l'étude des troubles des apprentissages en mathématiques. Recherches En Didactique Des Mathématiques, 40(2), 223-268.

[121] Peteers, F. & Ouvrier-Buffet, C. (2019). Diagnosis tools of dyscalculia – contribution of didactics of mathematics to numerical cognition. In Jankvist, U. T., Van den Heuvel-Panhuizen, M., & Veldhuis, M. (Eds.). (2019). Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11) (pp. 4664-4671). Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

[122] Petitfour, É. (2017). Outils théoriques d'analyse de l'action instrumentée, au service de l'étude de difficultés d'élèves dyspraxiques en géométrie. Recherches En Didactique Des Mathématiques, 37(2–3), 247–288.

[123] PISA 2015. Les défis du système éducatif français et les bonnes pratiques internationales (décembre 2016), OCDE.

[124] Quéré, P.V. (2017). French engineers' training and their mathematical needs in the workplace: Interlinking tools and reasoning. In T. Dooley & G. Gueudet (Eds.), Proceedings of the Tenth Congress of the European Mathematical Society for Research in Mathematics Education (pp. 2233–2240). Dublin, Ireland: DCU Institute of Education and ERME.

[125] Quéré, P.-V. (2019). Les mathématiques dans la formation des ingénieurs et sur leur lieu de travail : études et propositions (cas de la France). Thèse, Université de Bretagne Occidentale.

[126] Rasmussen, C. & Borba, M. (2014). The Teaching and Learning of Calculus – In memoriam Arnold Kirsch. ZDM, 46(4), 635-646.

[127] Rasmussen, C., Zandieh, M., King, K., & Teppo, A. (2005). Advancing Mathematical Activity: A Practice-Oriented View of Advanced Mathematical Thinking. Mathematical Thinking and Learning, 7(1), 51-73.

[128] Ravel, L. (2003). Des programmes à la classe : étude de la transposition didactique interne. Exemple de l'arithmétique en Terminale S spécialité. Thèse, Université Joseph Fourier, Grenoble I.

[129] Référentiel des compétences `https://www.education.gouv.fr/cid73215/le-referentiel-de-competences-des-enseignants-au-bo-du-25-juillet-2013.html`

[130] Repères et références statistiques, Enseignements. Formation. Recherche (2020). DEPP `https://www.education.gouv.fr/reperes-et-references-statistiques-2020-1316`

[131] Robert, A. (1998). Outils d'analyses des contenus mathématiques à enseigner au lycée et à l'université, Recherches en didactique des mathématiques, 18(2), 139-190.

[132] Rocha, K. M. (2017). Uses of Online Resources and Documentational Trajectories: the Case of Sésamath. In L. Fan, L. Trouche, S. Rezat, C. Qi,& J. Visnovska (Eds.), Research on Mathematics Textbooks and Teachers' Resources: Advances and issues. Springer.

[133] Rogalski, M. (2016). Revenir au sens de la notion de limite par certaines de ses raisons d'être : un chantier pour le début de l'analyse à l'université. In E. Nardi, C. Winsløw & T. Hausberger (Eds.), Proceedings of the First Conference of the International Network for Didactic Research in University Mathematics (INDRUM 2016, 31 March-2 April 2016) (pp. 133-142). Montpellier, France: University of Montpellier and INDRUM.

[134] Sabra, H. & Trouche, L. (2011). Collective design of an online math textbook: when Individual and collective documentation works meet, In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), Proceedings of the Seventh Congress of the European Mathematical Society for Research in Mathematics Education (pp. 2356-2366). Rzeszów, Poland: University of Rzeszów and ERME.

[135] Sayac, N. (2018). Summative assessment in mathematics at French primary school: A study based on a didactic approach. Classroom Assessment in Mathematics: Perspectives from around the Globe, In M. Burton, A. Cusi, D. R. Thompson & D. Wright (Eds.), (pp.159-178). New York: Springer.

[136] Selden, A. (2012) Transitions and proof and proving at tertiary level. In G. Hanna & M. de Villiers (eds), ICMI Study 19 Book: Proof and Proving in Mathematics Education (pp. 391-420). New-York : Springer.

[137] Selden, J. & Selden, A. (1995). Unpacking the logic of mathematics statements. Educational Studies in Mathematics, 29(2), 123-151.

[138] Shimizu, Y., Vithal, R. (2018). *School Mathematics Curriculum Reforms: Challenges, Changes and Opportunities. Proceedings of ICMI Study 24 Conference.* Tsukuba : University of Tsukuba. `https://drive.google.com/file/d/1xR0QOjI8X_mJb5mmhH-G_OqoMaFE54Gy/view`

[139] Socle commun de connaissances, de compétences et de culture (BO du 17 au 23 avril 2015) `http://cache.media.education.gouv.fr/file/17/45/6/Socle_commun_de_connaissances,_de_competences_et_de_culture_415456.pdf`

[140] Socle commun des connaissances et des compétences (Décret du 11 juillet 2006) http://cache.media.education.gouv.fr/file/51/3/3513.pdf

[141] Stratégie Mathématiques. https://www.education.gouv.fr/cid84398/strategie-mathematiques.html

[142] Tabchi, T. (2021). Relation entre enseignement et recherche dans le travail documentaire des enseignants-chercheurs – cas de l'enseignement de la théorie des graphes. Thèse, Université de Reims (France) et Université Libanaise (Liban).

[143] TIMSS 2019, 8e annnée. Cadre d'évaluation Mathématiques

[144] Trouche, L., Gueudet, G.,& Pepin, B. (2019). The "Resource" Approach to Mathematics Education. New York, NY: Springer.

[145] Trouche, L., Gueudet, G., & Pepin, B. (2020). Documentational approach to didactics. In S. Lerman (Ed.), Encyclopedia of Mathematics Education (2nd edition, pp. 237-247). Cham: Springer.

[146] Trouche, L., Rocha, K., Gueudet, G., & Pepin, B. (2020). Transition to digital resources as a critical process in teachers' trajectories: the case of Anna's documentation work. ZDM Mathematics Education, 52, 1243-1257, https://rdcu.be/b4txI doi 10.1007/s11858-020-01164-8

[147] Vandebrouck F. & Leidwanger S. (2016). Students' vizualisation of functions from secondary to tertiary level. In E. Nardi, C. Winsløw & T. Hausberger (Eds.), Proceedings of the First Conference of the International Network for Didactic Research in University Mathematics (INDRUM 2016, 31 March-2 April 2016) (pp. 153-162). Montpellier, France: University of Montpellier and INDRUM.

[148] Vergnaud, G. (1998). Toward a cognitive theory of practice, in A. Sierpinska & J. Kilpatrick (eds.), Mathematics education as a research domain: a search for identity, Kluwer Academic Publisher, Dordrecht, pp.227-241.

[149] Villani, C., & Torossian, C., (2018). 21 mesures pour l'enseignement des mathématiques. https://www.education.gouv.fr/cid126423/21-mesures-pour-l-enseignement-des-mathematiques.html

[150] Wang, C. (2019). An investigation of mathematics teachers' documentation expertise and its development in collectives: two contrasting cases in China and France. Ph.D. of ENS de Lyon and East China Normal University, retrained at: https://tel.archives-ouvertes.fr/tel-02275820

[151] Weber, K. (2008). How mathematicians determine if an argument is a valid proof. Journal for Research in Mathematics Education, 39(4), 431-459.

[152] Wenger, E. (1998). Communities of practice. Learning, meaning, identity. New York: Cambridge University Press.

[153] Winslow, C. (2013). ehiye Ubuz, Çiğdem Haser, Maria Alessandra Mariotti (Eds.), Proceedings of the 8th Congress of the European Society for Research in Mathematics Education, pp. 2476-2485.

[154] Winsløw, C., Gueudet, G., Hochmut, R. & Nardi, E. (2018). Research on University Mathematics Education. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger & K. Ruthven (eds.), Developing Research in Mathematics Education: Twenty Years of Communication, Cooperation and Collaboration in Europe (pp.60-74). London: Routledge.

[155] Winsløw, C., Biehler, R., Jaworski, B., Rønning, F. and Wawro, M. (2021) Education and professional development of University Mathematics Teachers . In V. Durand-Guerrier, R. Hochmuth, E. Nardi, C. Winsløw (eds) Research and Development in University Mathematics Education. Overview produced by the International Network for Didactic Researchin University Mathematics. pp.59-79, London: Routledge

[156] Yvain-Prébiski, S. (2018). Etude de la transposition à la classe de pratiques de chercheurs en modélisation mathématique dans les sciences du vivant. Analyse des conditions de la dévolution de la mathématisation horizontale aux élèves (Doctoral dissertation, Université Montpellier).

Acknowledgements, contributions

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Aurélien ALVAREZ, Martin ANDLER, Pierre ARNOUX, Michèle ARTIGUE, Anne CORTELLA, Christian DUHAMEL, Viviane DURAND-GUERRIER, Edwige GODLEWSKI, Ghislaine GUEUDET, Colette GUILLOPÉ, Simon MODESTE, Christian MERCAT, Louise NYSSEN, Cécile OUVRIER-BUFFET, Aviva SZPIRGLAS, Nicolas TOSEL, Stéphane VINATIER, Johan YEBBOU.

The English and Spanish translations benefited from the free version of the online automatic translator DeepL
`www.DeepL.com/Translator`

We thank
Isabelle ALIAGA, Claudia Gabriela REYES AVENDAÑO, Anne-Mercedes BELLIDO, Matias PAVEZ BRAVO, Anne-Marie CASTLE, Macarena FLORES GONZÁLEZ and Avenilde ROMO
for helping to proofread the English and Spanish versions.